PERFORMANCE ENHANCED BEAMFORMING ALGORITHMS FOR MIMO SYSTEMS

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ABSTRACT

The increasing capacity and quality demands for Multiple-Input-Multiple-Output (MIMO) services without a corresponding increase in RF spectrum allocation motivate the need for new techniques to improve spectrum utilization. Smart Antenna comprises of two functions viz. Angle Of Arrival (AOA) and Adaptive Beamforming (ABF). In this paper Beamforming algorithms namely Least Mean Square (LMS), Sign Error LMS (SE-LMS), Sign Data LMS (SD-LMS), Sign Sign LMS (SS-LMS) algorithms were simulated for various look directions and jammer configurations and their MSE characteristics and phase transients are compared. Performance of SD-LMS algorithm is studied by varying the step size. SS-Beamforming algorithm is also implemented on DSP kit TMS320C6713 and compared with simulation result.

Keywords: Adaptive Array Beamforming, LMS Algorithm, Sign Algorithms, MSE, MIMO.

INTRODUCTION

SMART antenna systems employing multiple antennas promises increased system capacity, extended radio coverage and improved quality of service through the ability to steer the antenna pattern in the direction of desired user while placing nulls at interferer locations [1]–[3].

Adding more antennas to the array gives higher angle resolution while steering the beam and more degrees of freedom in placing the nulls, but it results in increase of computational complexity and latency in calculating the weight vectors, which are used to process the received signals at the antennas. In Switched-beam approach a set of weight vectors are pre-calculated and stored for different angles, hence there is less computational complexity. In fully adaptive systems, however, a new weight vector is calculated adaptively with the change in the angle of the user and/or an
interferer. The adaptive approach, therefore, offers accurate tracing of the user angle at the cost of increased computational complexity.

The computational requirements of conventional LMS algorithm is high, therefore we need to devise methods to reduce complexity of beamforming algorithm without considerable degradation in performance. In this paper well known LMS algorithm is modified to make it suitable for high speed digital communication systems by reducing the complexity in the weight vector updation. The LMS algorithm is modified to obtain SE-LMS, SD-LMS and SS-LMS algorithms.

II. FORMULATION OF ADAPTIVE BEAMFORMING

![Uniform Linear Array](image)

Figure 1: Uniform Linear Array

A uniform linear array is as shown in *Figure (1)*, which consists of L equi-spaced omni-directional sensors with inter-element spacing of $\frac{\lambda}{2}$.

It receives M narrowband interference signals, one desired signal $s(n)$ and noise signal $n(n)$. The received data vector $x(n)$ is given by:

$$x(n) = a(\theta_o) s(n) + \sum_{i=1}^{M} a(\theta_i) \times i(n) + n(n) \quad (1)$$

Where, $a(\theta_o)$ is the desired steering vector and $a(\theta_i)$ is the steering vector for the $i^{th}$ interference signal [3].

If $w(n)$ is the complex weight then the output of a linear beamformer is:

$$y(n) = w(n)^T x(n) \quad (2)$$

$w(n)$ are usually estimated through the minimization of error $e(n)$ given by:

$$e(n) = s(n) - y(n) \quad (3)$$

III. LEAST MEAN SQUARE ALGORITHM (LMS)

The LMS algorithm is the most widely used adaptive beamforming algorithm, being employed in several communication applications. The LMS algorithm changes the weight vector...
$w(n)$ along the direction of the estimated gradient based on the steepest descent method. The weight vector updation for LMS algorithm is given by:

$$w(n + 1) = w(n) + \mu e(n) x^T(n)$$  \hspace{1cm} (4)$$

Where, $\mu$ is the step size controlling the convergence characteristics of beamforming algorithm given by:

$$0 \leq \mu \leq \frac{2}{\text{tr}(R_{xx})}$$  \hspace{1cm} (5)$$

$\text{tr}(R_{xx})$ is the trace of auto correlation matrix.

**IV. SIGN ERROR LMS ALGORITHM (SE-LMS)**

LMS algorithm is computationally efficient but the convergence rate is low and complexity is high. Therefore, the LMS algorithm is to be modified to achieve better convergence and reduced complexity by using SE-LMS.

The array weight coefficients of LMS are modified by applying sign operator to error $e(n)$ given by:

$$w(n + 1) = w(n) + \mu x(n) \text{sgn}(e(n))$$  \hspace{1cm} (6)$$

Where, $\text{sgn}(e(n))$ is given by

$$\text{sgn}(e(n)) = \begin{cases} 1 & e(n) > 0 \\ 0 & e(n) = 0 \\ -1 & e(n) < 0 \end{cases}$$  \hspace{1cm} (7)$$

The SE-LMS algorithm is also known as Least Mean Absolute Value (LMAV) algorithm. The sign error algorithm can be viewed as the result of applying two level quantizer to error $e(n)$.

**V. SIGN DATA LMS ALGORITHM (SD-LMS)**

This is similar to SE-LMS, instead of using the sign operator for error, the computational requirement of the LMS algorithm may be simplified by applying sign operator to the data as:

$$w(n + 1) = w(n) + \mu \text{sgn}(x(n)) e(n)$$  \hspace{1cm} (8)$$

Where, $\text{sgn}(x(n))$ is the sign of data vector given by:

$$\text{sgn}(x(n)) = \frac{x(n)}{|x(n)|}$$  \hspace{1cm} (9)$$

The disadvantage of SD-LMS algorithm is that it sometimes alters the direction of weight vectors.
VI. SIGN SIGN LMS ALGORITHM (SS-LMS)

In this algorithm we quantize both error and data by applying $\text{sgn}$ operator. The weight update equation is:

$$w(n+1) = w(n) + \mu \text{sgn}(x(n)) \text{sgn}(e(n))$$  \hspace{1cm} (10)

VII. SIMULATION RESULTS

The performance of all Beamforming algorithms stated has been studied by means of MATLAB simulation. For comparison purpose, result obtained with the conventional LMS algorithm is also presented.

For Simulation the following assumptions are considered

1. Mutual Coupling effects are zero between antenna elements
2. Distance between antenna elements is $\frac{\lambda}{2}$ an optimum value to avoid grating lobes
3. Look Direction and jammer directions have been determined aprior.
4. The propagation environment is stationary.
5. Number of array elements is 100.

CASE (A): Beamforming Result for LMS

Look Direction=45°
Interference Directions=10° and 30°.

![LMS Beamforming](image)

Figure (2): Polar Plot of LMS algorithm

From the Fig(2) it is clear that LMS algorithm is able to form the main beam in the look direction of 45° and nulls in the direction of interferers i.e 10° and 30°.
CASE (B) : Beamforming Result for SE-LMS

Look Direction=60°
Interference Directions=10°,20°,30° and 45°

![SE-LMS Beamforming](image)

**Figure (3):** Polar Plot of SE-LMS algorithm

From the Fig 3 it is clear that SE-LMS algorithm is able to form the main beam in the look direction of 60° and nulls in the direction of interferers i.e 10°,20°,30° and 45°.

CASE (C) : Beamforming Result for SD-LMS algorithm

Look Direction=30°
Interference Directions=10°,45°,55° and 60°

![SD-LMS Beamforming](image)

**Figure (4):** Polar Plot of SD-LMS algorithm

From the Fig (4) it is clear that SD-LMS algorithm is able to form the main beam in the look direction of 30° and nulls are diminishing in the direction of interferers. Hence SD-LMS has an advantage of completely blocking the jammer directions.

CASE (D) : Beamforming Result for SS-LMS

Look Direction=70°
Interference Directions=10°,20°,30° and 60°
Figure (5): Polar Plot of SS-LMS algorithm

From the Fig (5) it is clear that SS-LMS algorithm is able to form the main beam in the look direction of $70^0$ and nulls are in the direction of interferers.

CASE (E): Weight Vector Computation

The performance of various algorithms is compared in terms of phase variations applied to individual array elements by performing 100 runs. LMS algorithm has the larger variations in phase shifts whereas sign algorithms have lesser variation’s in phase as depicted in simulation curves of Fig (6).

CASE(F) : Effect of Change in Step Size on MSE

Figure(7): MSE of SD-LMS algorithm
The performance of SD-LMS is studied by varying the step size. From the simulation curve of Fig (7) it is clear that as the step size increases the algorithm takes more iterations to converge.

**CASE(G): Effect of Change in Step Size on weight magnitude**

![Figure 8: Step Size Variation Effects](image)

The performance of SD-LMS algorithm is studied by varying the step size. As the step size is increased the weight vector magnitude has large amount of variations. Hence it is good to choose smaller step size for better performance as shown in simulation curve of Figure(8).

**CASE (G): Error Vector Magnitude (EVM) measurement of Beamforming algorithms**

For a complex signal, it is also convenient to make use of the Error Vector Magnitude (EVM) as an accurate measure of any distortion introduced by the adaptive scheme on the received signal [8].

The EVM is given by:

\[
EVM_{\text{rms}} = \sqrt{\frac{1}{KP_0} \sum_{j=1}^{K} |S_r(j) - S_t(j)|^2}
\]

Where, \(K\) is the number of observations used, \(S_r(j)\) is the normalized \(j\)th output of the beamformer and \(S_t(j)\) is the \(j\)th transmit signal. \(P_0\) is the normalized transmit signal power.

\(K = \text{Number of observations} = 100.\)

**Table (1): Simulation Results of EVM**

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>EVM</th>
</tr>
</thead>
<tbody>
<tr>
<td>LMS</td>
<td>0.9870</td>
</tr>
<tr>
<td>SE-LMS</td>
<td>0.9760</td>
</tr>
<tr>
<td>SD-LMS</td>
<td>0.9582</td>
</tr>
<tr>
<td>SS-LMS</td>
<td>0.9844</td>
</tr>
</tbody>
</table>

From the MATLAB simulation results of TABLE(1) it is clear that the EVM of SD-LMS algorithm is less as compared to other algorithms.
CASE (H): Computation Complexity & Execution Speed

Computation complexity is also a good performance index to measure the efficiency of Beamforming algorithms. If $L$ is number of array elements and $N$ is number of iterations.

Table (2): Computation Complexity of Beamforming

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Number of Additions</th>
<th>Number of Multiplications</th>
</tr>
</thead>
<tbody>
<tr>
<td>LMS</td>
<td>$N(L+1)$</td>
<td>$N(L+2)$</td>
</tr>
<tr>
<td>SE-LMS</td>
<td>$N$</td>
<td>$N$</td>
</tr>
<tr>
<td>SD-LMS</td>
<td>$N(2L+4)$</td>
<td>$N(2L-2)$</td>
</tr>
<tr>
<td>SS-LMS</td>
<td>$N$</td>
<td>$NL$</td>
</tr>
</tbody>
</table>

From the TABLE(2) SE-LMS requires least number of multiplication’s and additions followed by SS-LMS where as conventional LMS requires large amount of multiplications.

Table (3): Execution Speed of Beamforming

<table>
<thead>
<tr>
<th>Beamforming algorithm</th>
<th>Execution Time in seconds</th>
</tr>
</thead>
<tbody>
<tr>
<td>L.M.S</td>
<td>0.8541</td>
</tr>
<tr>
<td>SD-LMS</td>
<td>0.3084</td>
</tr>
<tr>
<td>SE-LMS</td>
<td>0.2297</td>
</tr>
<tr>
<td>SS-LMS</td>
<td>0.4780</td>
</tr>
</tbody>
</table>

From the TABLE (3) it is clear that SE-LMS takes least amount of execution time followed by SD-LMS as compared to conventional LMS.

VIII. IMPLEMENTATION RESULTS ON TMSC3206713

In this section graphs for real and imaginary phase shifts obtained using DSP kit for SS-LMS algorithm are presented.

Table (4): Input to SS-LMS Beamformer

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Number of Array Elements</th>
<th>Look Direction</th>
<th>Jammer Directions</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS-LMS</td>
<td>8</td>
<td>30$^\circ$</td>
<td>[10$^\circ$,20$^\circ$,70$^\circ$]</td>
</tr>
</tbody>
</table>

Figure (9): DSP kit Output for real array weights calculation using SS-LMS Algorithm
Fig(9) gives the real part of array weights calculated on DSP kit. ‘w(i)’ indicates array weight, i is the index corresponding to antenna element.

Figure (10): DSP kit Output for imaginary array weights calculation using SS-LMS Algorithm

Figure (10) gives the imaginary part of array weights calculated on DSP kit.

IX. COMPARISON OF SIMULATION AND IMPLEMENTATION RESULTS

Figure (11): Comparison of MATLAB and DSP kit Result for SS-LMS

Figure (11) provides the comparison of weight vector obtained using MATLAB and DSP kit. Both the weight vector’s almost Converges.

X. CONCLUSION

LMS algorithm is modified to obtain sign algorithms of beamforming by applying the sgn operator to error, data and both.

It is shown that sign algorithms have reduced Error Vector Magnitude, complexity and execution speed as compared to conventional LMS algorithm. Simulation curves also reveal that the phase transients of LMS are higher as compared to Sign algorithms. Performance of SD-LMS is simulated by varying step size.

REFERENCES


