COMPARISON OF THREE METHODS TO SEPARATE WAVES IN THE PROCESSING OF LONG-TIME HOPKINSON BAR EXPERIMENTS

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ABSTRACT

In order to use the split Hopkinson bar setup in the intermediate strain rate range, several wave separation methods have been proposed in the literature. In this work, three wave separation methods are compared using numerical simulation: Casem-Fourney-Chang (CFC), Jacquelin-Hamelin (JH) and Bussac-Collet-Gary-Othman (BCGO) methods. It is observed that the CFC and JH methods are more accurate than the two-strain and three-strain wave separation methods. The four-strain wave separation method does better than CFC and JH methods, where also, JH method is slightly better than the CFC method. Moreover, the BCGO method error decreases with increasing number of strain measurements.

Keywords: Hopkinson bar, Intermediate strain rate, Kolsky bars, Wave dispersion, Wave separation.

I. INTRODUCTION

Intermediate strain-rate (1-200/s) mechanical testing of materials is a main concern in several engineering applications [1-4]. The Hopkinson-Kolsky bar technique is mostly used at high strain-rates (500-5000/s), because of the limitation on the test duration. This limitation is induced by the superposition of waves propagating in opposite directions.

Since the pioneer works of Lundberg & Henchoz [5] and Yanagihara [6], several wave separation techniques with no limit on the test duration have been proposed [7-17]. The reader is referred to Othman et al. [7] for a critical review of these techniques. Mostly, wave separation methods were applied to extend Hopkinson-Kolsky bar machine to the intermediate strain-rate range.
[8-11]. Othman et al. [7] have also applied a wave separation method to an elastic bar replacing the piezo-electric force sensor in a servo-hydraulic machine.

Wave separation techniques applied to elastic and viscoelastic bars can be considered as long-duration dynamic force measurement. Hence, they have multiple applications in impact engineering. From the critical review of Othman et al. [7], it seems that three methods are less sensitive to noise than the others. These methods are: CFC method [12], JH method [13] and BCGO method [14-15]. In this paper, we aim at comparing these three wave separation techniques by using numerical simulations.

II. METHODOLOGY

The goal of this work is to compare three CFC, JH and BCGO wave separation methods. To this purpose, we consider a long viscoelastic bar (Fig. 1). The bar left end is impacted by a striker whereas the right end is free. The three wave separation methods use strain and/or velocity measurements.

As a first step, strain and velocity measurements are simulated assuming 1D wave propagation in the bar. However, 3D geometrical effects on wave dispersion are taken into account [18]. The measurement positions are chosen randomly. One strain and one velocity measurements, in the same cross-section, are considered for the CFC method. For the JH method, three strain measurements are simulated. The BCGO method can use an infinite number of strain measurements. In this work, we study the BCGO method using 2, 3, 4, 5, 10, 20, 50 and 100 strain measurements. Numerical simulations are carried out during a period of time $T$ equal to 10 ms. Firstly, the Fourier transforms of strain and velocity are calculated. Then the inverse Fourier transform is used to deduce strain and velocity in terms of time. More precisely, the Fourier transform of the strain is given by

$$\tilde{\xi}(x, \omega) = \sum_{m=0}^{M_0} \tilde{\theta}(\omega) e^{-i\xi(\omega)[(2m+1)L+x]} + \sum_{m=0}^{M_0} -\tilde{\theta}(\omega) e^{-i\xi(\omega)[(2m+1)L-x]},$$

(1)

where $\tilde{\theta}(\omega)$ is the Fourier transform of the wave generate by the striker impact at the left bar end, $\xi(\omega)$ is the wave dispersion relation, $L$ is the bar length, $x$ is the strain measurement position, $\omega$ is the angular frequency and $M_0$ is the number of wave round-trips during the time $T$. Similarly, the Fourier transform of the velocity reads

$$\tilde{\nu}(x, \omega) = \frac{\omega}{\xi(\omega)} \sum_{m=0}^{M_0} \tilde{\theta}(\omega) e^{-i\xi(\omega)[(2m+1)L+x]} + \sum_{m=0}^{M_0} \tilde{\theta}(\omega) e^{-i\xi(\omega)[(2m+1)L-x]},$$

(2)
In order to simulate experimental noise, a numerically build Gaussian noise is added to each strain or velocity signal. These noises are constructed separately. Therefore, they can be assumed two-by-two independent.

In this work, we consider that the bar is 4-m long and 40 mm in diameter. It is supposed to be made of aluminum (Young’s modulus \( E = 70 \text{ GPa} \), Poisson’s ratio \( v = 0.34 \) and density \( \rho = 2800 \text{ kg/m}^3 \)). The wave dispersion equation is obtained by solving the Pochhammer-Chree equation [19,20]. The calculated strain and velocity signals are sampled at a sampling frequency of 100 MHz.

The second step consists in calculating the force at the free end, which should be equal to zero, by applying the three different wave separation methods to the appropriate strain and/or velocity measurements. The reader is referred to Refs. [12-14] for more details on how these methods work.

Let \( F_{\mu}^r = F_{\mu}(r \Delta t) \) be the force at the free bar end at the time \( r \Delta t \) (\( \Delta t \) is the sampling step). The subscript \( \mu \) refers to the used method, thus, \( \mu \) can be replaced by \( \text{CFC}, \text{ JH}, \text{ or BCGO}_J \) for CFC method, JH method and J-strain measurements BCGO method, respectively. The error of the method \( \mu \) is defined by

\[
\zeta_{\mu}^a = \frac{\| F_{\mu}^r \|_a}{F_0},
\]

where \( \| . \|_a \) is a norm and \( F_0 \) is a reference force. Two norms are considered here the Euclidian norm \( \| . \|_2 \) and the max norm \( \| . \|_{\infty} \). For a vector \( Y = (y_r)_{r=0,...,R} \), the two norms read

\[
\| Y \|_2 = \sqrt{\sum_{r=0}^{R} (y_r)^2},
\]

and

\[
\| Y \|_{\infty} = \max_r (y_r).
\]

The reference force \( F_0 \) is chosen to be the maximum force induced by the striker impact at the left bar end.

By running the first and second step, we obtain six error values, two for each method. The results depend on the considered strain and/or velocity simulated measurement positions which are chosen randomly. It depends also on the numerical noise added to the perfect strain and velocity measurements. Therefore, steps 1 and 2 are run for one thousand times. Each time six error values are obtained. At the end of the run, an average value is calculated: \( \langle \zeta_{\mu}^a \rangle \). \( \langle \zeta_{\mu}^a \rangle \) is the average error of the method \( \mu \) calculated using the norm \( \| . \|_a \).

III. RESULTS

The errors obtained by the CFC, JH and BCGO methods are depicted in Fig. 2. For the BCGO\(_J\) method, eight values of \( J \) are considered: \( J = 2; 3; 4; 5; 10; 25; 50; 100 \). Even though, it is hard to cement 10 strain measurements or more on a bar, these values are only considered to check the J-convergence of the BCGO\(_J\) method. Moreover, we can imagine the use of full-field strain measurement techniques. In that case a high number of strain measurements can be recorded.

Excepting the BCGO\(_2\), all methods give quite accurate results as they have average maximum errors lower than 8% (Fig. 2 (b)) and average \( \| . \|_2 \) errors below 2% (Fig. 2(a)). The CFC and JH methods give better results than the two- and three-strain BCGO wave separation method. The JH method is slightly better than the CFC. The BCGO method is more accurate for \( J \geq 4 \). The errors of
the BCGO method decrease with increasing \( J \). These errors have an exponential behavior for important values of strain measurements: \( J \geq 5 \). Precisely,

\[
\langle z^a_{BCGO_j} \rangle \propto e^{-j}. \tag{6}
\]

It is worth noticing that BCGO\(_2\) gives highly dispersive results, following in decreasing order: BCGO\(_3\), BCGO\(_4\), JH, CFC than BCGO\(_{\geq5}\). The dispersion is most probably due to the variation of the measurements positions. These positions can influence the poles in the solutions of the BCGO wave separation methods. As errors are amplified in the neighborhood of these poles, this can explain why the BCGO method gives more dispersive results. Even though, the BCGO\(_4\) is more dispersive, it gives, in average, better results than JH and CFC methods.

**IV. CONCLUSION**

Assuming one-dimensional wave propagation and considering the 3D wave dispersion effects, strain and velocity measurements, induced by a striker impact, were simulated in a free-ended elastic bar. Subsequently, a numerically Gaussian noise was added to the strain and velocity measurements to be close to experimental situation. The noise measurements were then processed by three wave separation methods in order to recover the force at the free end, which should be equal zero. Thus, we were able to define and calculate an error for each wave separation method.

It was observed that the CFC and JH wave separation methods lead to more accurate results than the two-strain and three-strain BCGO method. However, the four-strain wave separation method gives better accuracy than the CFC and JH methods. Comparing these last two, JH method is slightly better than CFC method.

It is also reported that the increase of the number of strain measurement improves the accuracy of the BCGO wave separation method. Actually, increasing the number of measurements increases the redundancy.
REFERENCES


Fig. 2: Comparison of errors obtained by the three wave separation method: (a) $\langle \xi^2 \rangle$ and (b) $\langle \xi^\infty \rangle$


