A NUMERICAL STUDY OF THREE-DIMENSIONAL DARCY-BRINKMAN-FORCHHEIMER (DBF) MODEL IN A INCLINED RECTANGULAR POROUS BOX

Dr. R. P. Sharma *; Dr. R. V. Sharma **

*Dept. of Mechanical Engineering, Birla Institute of Technology, Mesra, Ranchi, 835215 India. E-mail: rpsharmabit123@gmail.com

**Dept. of Mechanical Engineering, National Institute of Technology, Jamshedpur, 831014 India. E-mail: ramvinoysharma@yahoo.com

ABSTRACT

In this paper, numerical studies on three-dimensional natural convection in an inclined differentially heated porous box employing Darcy-Brinkman-Forchheimer flow model are presented. The relative effects of inertia and viscous forces on natural convection in porous media are examined. The system is characterized by Rayleigh number (Ra), two aspect ratios (AR_Y, AR_Z), Darcy number (Da), ratio of Forchheimer number to Prandtl number (Fc/Pr) and angle of inclination (\(\phi\)). Numerical solutions have been obtained employing the SAR scheme for Ra = 1000, AR_Y =1.0, AR_Z =1.0, \(10^{-5}<\text{Da}<10^{-2}\), \(10^{-5}<\text{Fc/Pr}<10^{-2}\) and \(-60^\circ<\phi<60^\circ\). The reduction in average Nusselt number due to Brinkman viscous terms and Forchheimer non-linear inertial terms become significant when \(\text{Da}>10^{-5}\) and \(\text{Fc/Pr}>10^{-5}\) respectively. There exists a critical angle of inclination of the porous box at which the average Nusselt number becomes maximum. The critical angle of inclination for Darcy flow description is 30\(^\circ\) where as for non-Darcy flow (D-B, D-F, D-B-F) description, the critical angle of inclination is \(-30^\circ\).

1.0 INTRODUCTION

Studies on natural convection heat transfer in porous media employing non-Darcy extensions to describe the fluid flow which include Forchheimer non-linear inertial terms, convective terms and Brinkman viscous terms, have been reported in the literature. Chan et al. [1] included the viscous terms due to Brinkman as an extension to the Darcy equation to describe the fluid flow. They concluded that the average Nusselt number shows a maximum when the aspect ratio is around unity. Poulikakos and Bejan [2] employed Forchheimer extended Darcy flow model and obtained boundary layer solutions for a tall cavity and the agreement with the numerical results for Ra = 2000 and AR = 2 has been found to be good. Based on these studies, Poulikakos and Bejan classified the flow regimes as Darcy, intermediate and non-Darcy. Tong and Subramaniam [3] developed boundary layer solutions to Brinkman extended Darcy flow model based on the
modified Oseen technique and the flow field is found to be governed by a new parameter. Tong and Subramaniam found that a pure Darcy analysis is applicable when the parameter defined in their study is less than $10^{-4}$. A numerical study based on the Forchheimer–Brinkman-extended Darcy equation of motion has been reported by Beckermann et al., [9]. Lauriat and Prasad [5] examined the relative importance of inertia and viscous forces via the Darcy–Brinkman-Forchheimer solutions for a differentially heated vertical porous cavity. The results indicate that there exists an asymptotic convection regime where the heat transfer rate is independent of permeability of the porous matrix. Kaneko et al., [11] in an experimental investigation on natural convection in liquid saturated confined porous medium has shown that the mode and intensity or convective motions are affected by the angle of inclination of the medium and certain properties of the saturating fluid. Sen et al., [10] investigated the multiplicity of steady states in natural convection within an inclined porous material with parallel conductive isotherms. The different steady states are obtained analytically for unicellular convection in thin rectangular porous layers with uniform heating and cooling through opposite walls. The basis of the analytical approximation is an assumption of parallel flow over in large portion of the layer. The two cases of heat fluxes through side and end walls are both calculated and are seen to share some qualitatively similar features. At sub-critical Rayleigh numbers only one steady state exits for any tilt angle. For higher Rayleigh numbers and bottom heating, however, multiple steady states exist, some of which are unstable. Numerical confirmation of the stable analytical results is also presented. Vasseur et al., [7] studied analytically and numerically the thermally driven flow in a thin, inclined, rectangular cavity filled with a fluid saturated porous layer. Moya et al., [8] analyzed two-dimensional natural convective flow in a tilted rectangular porous material saturated with fluid by solving numerically the mass, momentum and energy balance equations using Darcy's law and the Boussinesq approximation. Isothermal boundary conditions are considered, where two opposite walls are kept at constant but different temperatures and the other two are thermally insulated. The external parameters considered are the tilt angle, the aspect ratio and the Darcy-Rayleigh number. Three main convective modes are found. Conduction, single and multiple cell convection and their features described in detail. Local and global Nusselt numbers are presented as functions of the external parameters.

2.0  MATHEMATICAL MODELLING

2.1  Governing Equation

Fig. 1 Physical model and co-ordinate system
The physical model is shown in fig. 1 is a parallelepiped box of length L, width B and height H filled with fluid saturated porous medium. Governing equations for natural convection in the porous box comprising of conservation of mass, momentum and energy are as follows:

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \]  

(1)

\[ \frac{\mu}{K} u + K' \rho |u| u = \left( \frac{\partial p}{\partial x} + \rho g \sin \phi \right) + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \]  

(2)

\[ \frac{\mu}{K} v + K' \rho |v| v = \left( \frac{\partial p}{\partial y} + \rho g \cos \phi \right) + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \]  

(3)

\[ \frac{\mu}{K} w + K' \rho |w| w = \left( \frac{\partial p}{\partial z} \right) + \mu \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) \]  

(4)

\[ \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) \]  

(5)

Governing equations are rendered dimensionless introducing the following non-dimensional variables:

\[ X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad Z = \frac{z}{L} \]

\[ U = \frac{u}{\alpha / L}, \quad V = \frac{v}{\alpha / L}, \quad W = \frac{w}{\alpha / L} \]

\[ \theta = \frac{T - T_c}{T_h - T_c}, \quad \rho = \frac{\rho}{\rho_m}, \quad \beta = \frac{\beta}{\rho / \alpha / K} \]  

(6)

The dimensionless conservation of mass, momentum and energy are as follows:

\[ \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} + \frac{\partial W}{\partial Z} = 0 \]  

(7)

\[ U + \frac{F_c}{Pr} |V| U = - \frac{\partial P}{\partial X} - \frac{Ra}{\beta T_c} \sin \phi + Da \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} + \frac{\partial^2 U}{\partial Z^2} \right) \]  

(8)

\[ V + \frac{F_c}{Pr} |V| V = - \frac{\partial P}{\partial Y} - \frac{Ra}{\beta T_c} \cos \phi + Da \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} + \frac{\partial^2 V}{\partial Z^2} \right) \]  

(9)

\[ W + \frac{F_c}{Pr} |V| W = \frac{\partial P}{\partial Z} + Da \left( \frac{\partial^2 W}{\partial X^2} + \frac{\partial^2 W}{\partial Y^2} + \frac{\partial^2 W}{\partial Z^2} \right) \]  

(10)

\[ U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} + W \frac{\partial \theta}{\partial Z} = \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} + \frac{\partial^2 \theta}{\partial Z^2} \]  

(11)
\[ \rho = 1 - \beta \Delta T (0-0.5) \]  

(12)

Non-dimensional parameters \( \text{Ra} \), the Rayleigh number, \( \text{Fc} \), the Forchheimer number, \( \text{Pr} \), the Prandtl number and \( \text{Da} \), the Darcy number are defined by,

\[ Ra = \frac{K g L \beta \Delta T}{\nu \alpha} \]  

(13)

\[ Fc = \frac{K'}{L} \]  

(14)

\[ Pr = \frac{v}{\alpha} \]  

(15)

\[ Da = \frac{K}{L^2} \]  

(16)

Boundary conditions given by Eqs. (2.7) to (2.9) in dimensionless form become,

**Hydrodynamic Boundary Conditions**

(i) Without Brinkman terms

\[ U = 0 \quad \text{at} \quad X = 0,1 \quad \text{for} \quad 0 \leq Y \leq AR_Y \quad \text{and} \quad 0 \leq Z \leq AR_Z \]

\[ V = 0 \quad \text{at} \quad Y = 0, \ AR_Y \quad \text{for} \quad 0 \leq X \leq 1 \quad \text{and} \quad 0 \leq Z \leq AR_Z \]  

(17)

\[ W = 0 \quad \text{at} \quad Z = 0, \ AR_Z \quad \text{for} \quad 0 \leq X \leq 1 \quad \text{and} \quad 0 \leq Y \leq AR_Y \]

(ii) With Brinkman terms

\[ U = V = W = 0 \quad \text{at} \quad X = 0,1 \quad \text{for} \quad 0 \leq Y \leq AR_Y \quad \text{and} \quad 0 \leq Z \leq AR_Z \]

\[ U = V = W = 0 \quad \text{at} \quad Y = 0, \ AR_Y \quad \text{for} \quad 0 \leq X \leq 1 \quad \text{and} \quad 0 \leq Z \leq AR_Z \]  

(18)

\[ U = V = W = 0 \quad \text{at} \quad Z = 0, \ AR_Z \quad \text{for} \quad 0 \leq X \leq 1 \quad \text{and} \quad 0 \leq Y \leq AR_Y \]

**Thermal Boundary Conditions**

\[ \theta = 0 \quad \text{at} \quad X = 0 \quad \text{for} \quad 0 \leq Y \leq AR_Y \quad \text{and} \quad 0 \leq Z \leq AR_Z \]

\[ \theta = 1 \quad \text{at} \quad X = 1 \quad \text{for} \quad 0 \leq Y \leq AR_Y \quad \text{and} \quad 0 \leq Z \leq AR_Z \]  

(19)

\[ \frac{\partial \theta}{\partial Y} = 0 \quad \text{at} \quad Y = 0, \ AR_Y \quad \text{for} \quad 0 \leq X \leq 1 \quad \text{and} \quad 0 \leq Z \leq AR_Z \]

\[ \frac{\partial \theta}{\partial Z} = 0 \]

at \( Z = 0, \ AR_Z \quad \text{for} \quad 0 \leq X \leq 1 \quad \text{and} \quad 0 \leq Y \leq AR_Y \)

Where \( AR_Y \), the vertical aspect ratio and \( AR_Z \), the horizontal aspect ratio are defined as

\[ AR_Y = \frac{H}{L} \]  

(20)

\[ AR_Z = \frac{B}{L} \]  

(21)
Vector-Potential Formulation

Eliminating pressure term from the momentum equations (2.12)-(2.14),

\[
\begin{align*}
\Omega_x &+ \frac{Fc}{Pr} \left[ W \frac{\partial |V|}{\partial Y} - V \frac{\partial |V|}{\partial Z} + |V| \frac{\partial \Omega_x}{\partial \Omega} \right] = -Ra \frac{\partial \theta}{\partial Z} \cos \phi + Da \nabla^2 \Omega_x, \\
\Omega_y &+ \frac{Fc}{Pr} \left[ U \frac{\partial |V|}{\partial Z} - W \frac{\partial |V|}{\partial X} + |V| \frac{\partial \Omega_y}{\partial \Omega} \right] = Ra \frac{\partial \theta}{\partial Z} \sin \phi + Da \nabla^2 \Omega_y, \\
\Omega_z &+ \frac{Fc}{Pr} \left[ V \frac{\partial |V|}{\partial X} - U \frac{\partial |V|}{\partial Y} + |V| \frac{\partial \Omega_z}{\partial \Omega} \right] = -Ra \left( -\frac{\partial \theta}{\partial Y} \sin \phi - \frac{\partial \theta}{\partial X} \cos \phi \right) + Da \nabla^2 \Omega_z.
\end{align*}
\]

Energy equation given by Eq. (2.15) can be rewritten as,

\[
U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} + W \frac{\partial \theta}{\partial Z} = \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} + \frac{\partial^2 \theta}{\partial Z^2}.
\]

The velocity components \( U, V \) and \( W \) are related to the components of the vector-potential by,

\[
\begin{align*}
U &= \frac{\partial \Psi_x}{\partial Y} - \frac{\partial \Psi_y}{\partial Z}, \\
V &= \frac{\partial \Psi_y}{\partial Z} - \frac{\partial \Psi_z}{\partial X}, \\
W &= \frac{\partial \Psi_z}{\partial X} - \frac{\partial \Psi_x}{\partial Y}.
\end{align*}
\]

Boundary Conditions on Vector Potential \((\Psi)\)

Boundary condition on vector-potential \((\Psi)\) due to Hirasaki and Hellums [109] are given by,

\[
\frac{\partial \Psi_x}{\partial X} = \Psi_y = \Psi_z = 0
\]

at \( X = 0, 1 \)

\[
\frac{\partial \Psi_y}{\partial Y} = \Psi_z = \Psi_x = 0
\]

at \( Y = 0, AR_Y \)

\[
\frac{\partial \Psi_z}{\partial Z} = \Psi_x = \Psi_y = 0
\]

at \( Z = 0, AR_Z \)

(28)
Boundary Conditions on Vorticity-Vector ($\mathbf{\Omega}$)

(i) Vorticity vector ($\mathbf{\Omega}$) at the walls for no-slip condition due to Aziz and Hellums [110] are given by,

$$
\Omega_x = 0; \quad \Omega_y = -\frac{\partial W}{\partial X}; \\
\Omega_z = \frac{\partial V}{\partial X}
$$

at $X = 0, 1$

$$
\Omega_x = \frac{\partial W}{\partial Y}; \\
\Omega_y = 0; \quad \Omega_z = -\frac{\partial U}{\partial Y}
$$

at $Y = 0, AR_Y$

$$
\Omega_x = -\frac{\partial V}{\partial Z}; \quad \Omega_y = \frac{\partial U}{\partial Z}; \\
\Omega_z = 0
$$

at $Z = 0, AR_Z$ (29)

(ii) Vorticity vector ($\mathbf{\Omega}$) at the walls when velocity slip is allowed are given by,

$$
\Omega_x = \frac{\partial W}{\partial Y} - \frac{\partial V}{\partial Z}; \quad \Omega_y = -\frac{\partial W}{\partial X}; \quad \Omega_z = \frac{\partial V}{\partial X}
$$

at $X = 0, 1$

$$
\Omega_x = -\frac{\partial W}{\partial Y}; \quad \Omega_y = \frac{\partial U}{\partial Z} - \frac{\partial W}{\partial X}; \quad \Omega_z = -\frac{\partial U}{\partial Y}
$$

at $Y = 0, AR_Y$

$$
\Omega_x = -\frac{\partial V}{\partial Z}; \\
\Omega_y = \frac{\partial U}{\partial Z}; \\
\Omega_z = \frac{\partial V}{\partial X} \frac{\partial U}{\partial Y}
$$

at $Z = 0, AR_Z$ (30)

Boundary conditions on temperature ($\theta$) are the same as given by Eq (19). The average Nusselt number based on the characteristic length, $L$ of the box is defined as,

$$
Nu = \frac{\bar{h}L}{k}
$$
The average Nusselt number at \( X = 0 \) and \( X = 1 \) is obtained by numerical integration according to,

\[
Nu_h = \frac{1}{AR_y AR_Z} \int_0^{AR_y} \int_0^{AR_Z} \left. \frac{\partial \theta}{\partial X} \right|_{z=0} dY dz
\]  

(32)

\[
Nu_c = \frac{1}{AR_y AR_Z} \int_0^{AR_y} \int_0^{AR_Z} \left. \frac{\partial \theta}{\partial X} \right|_{z=1} dY dZ
\]  

(33)

In order to obtain the numerical solution of the above equations along with boundary conditions, the Successive Accelerated Replacement (SAR) scheme has been employed and results are obtained.

The average Nusselt number at \( X = 0 \) and \( X = 1 \) is obtained by numerical integration according to,

\[
Nu_h = \frac{1}{AR_y AR_Z} \int_0^{AR_y} \int_0^{AR_Z} \left. \frac{\partial \theta}{\partial X} \right|_{z=0} dY dZ
\]  

(34)

\[
Nu_c = \frac{1}{AR_y AR_Z} \int_0^{AR_y} \int_0^{AR_Z} \left. \frac{\partial \theta}{\partial X} \right|_{z=1} dY dZ
\]  

(32)

**RESULTS & DISCUSSION**

It can be seen from Fig. 2 that for the Darcy-Brinkman (D-B) and Darcy-Brinkman-Forchheimer (D-B-F) flow model, the no-slip boundary conditions are satisfied i.e. \( V = 0 \) at \( X = 0 \) and \( X = 1 \). For Darcy and Darcy-Forchheimer flow description, the no-slip boundary condition is not satisfied i.e \( V \neq 0 \) at \( X = 0 \) and \( X = 1 \). Also due to inertial and boundary effects the flow velocity decreases. In the core region, there is almost no flow. Boundary layer formation can be easily seen at the hot and cold walls.

Variation of average Nusselt number with angle of inclination is shown in Fig. 7.9. The average Nusselt number (Nu) values is higher at negative angle of inclination (\( \phi \)) and the critical angle of inclination is \(-30^\circ\).
Fig. 2. Variation of vertical velocity component ($\bar{V}$) at $Y = 0.5$ with $X$ for $Ra = 1000$, $AR_y = AR_z = 1.0$, $Da = 10^{-2}$ $F_{c/Pr} = 10^{-2}$ and $\phi = -30^\circ$.

The physical reason/mecanism can be explained on the basis of flow field shown in fig. 3 for $\phi = -30^\circ$. It can be seen from the above plots that for negative angle of inclination, there is a strong natural convection and multicellular flow is more pronounced for negative angle of inclination. The multicellular flow augments the heat transfer based on the physical ground that single cell heat transfer, having less heat transfer channels, be smaller than that due to multiple cells.

Fig. 3. Iso-vector potential ($\bar{\Psi}_x$) for $Ra = 1000$, $AR_y = AR_z = 1.0$, $Da = 10^{-2}$ $F_{c/Pr} = 10^{-2}$ and $\phi = -30^\circ$. 

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The relative effects of Da and Fc/Pr on average Nusselt number demonstrated in a comprehensive Table 7.3. For a given Darcy number as Fc/Pr decreases, the average Nusselt number value increases. This increase is more at low Darcy number. For Fc/Pr \( \leq 10^{-5} \), the effects of Fc/Pr is negligible. For a given Fc/Pr as Da decreases, the average Nusselt number increases. For Da \( \leq 10^{-5} \), the effect of Da is negligible.

![Fig. 7.9 Variation of Nu with \( \phi \) for Ra = 1000, AR\(_Y\) = AR\(_Z\) = 1.0 and Da = 10\(^{-4}\)](image)

Table 7.3: Comparison of average Nusselt number of D, D-B, D-F and D-B-F flow models for Ra = 1000, AR\(_Y\) = AR\(_Z\) = 1.0 and \( \phi = 0^\circ \).

<table>
<thead>
<tr>
<th>Fc/Pr</th>
<th>10(^{-2})</th>
<th>10(^{-3})</th>
<th>10(^{-4})</th>
<th>10(^{-5})</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Da</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10(^{-2})</td>
<td>3.86</td>
<td>3.98</td>
<td>3.99</td>
<td>3.99</td>
<td>3.99</td>
</tr>
<tr>
<td>10(^{-3})</td>
<td>6.12</td>
<td>6.85</td>
<td>6.95</td>
<td>6.95</td>
<td>6.95</td>
</tr>
<tr>
<td>10(^{-4})</td>
<td>7.67</td>
<td>9.63</td>
<td>10.08</td>
<td>10.14</td>
<td>10.14</td>
</tr>
<tr>
<td>10(^{-5})</td>
<td>8.46</td>
<td>11.60</td>
<td>12.63</td>
<td>12.77</td>
<td>12.78</td>
</tr>
<tr>
<td>0</td>
<td>8.48</td>
<td>12.07</td>
<td>13.49</td>
<td>13.71</td>
<td>13.72</td>
</tr>
</tbody>
</table>

### 7.3 CONCLUSIONS

Numerical solutions to the equations governing natural convection heat transfer in an inclined porous box have been obtained employing SAR scheme. The flow description is within the framework of Darcy-Brinkman-Forchheimer (D-B-F) flow model. The strength of free convection for the D-B-F flow model is lower than that of D-B and D-F flow models due to presence of both inertia effects and no-slip boundary conditions. The strength of free convection is highest at \( \phi = -30^\circ \). The critical angle of inclination for D-B-F flow model is \(-30^\circ\) irrespective of all other parameters. For Da \( < 10^{-5} \) and Fc/Pr \( < 10^{-5} \), the D-B-F flow model reduces to simple Darcy model. The three-dimensional effects are pronounced for AR\(_Z\) \( < 1 \). The average Nusselt number for 3-D is lower than that of 2-D system. The effect of Da is more pronounced than that of
REFERENCES