ANALYSIS OF POROUS ELLIPTICAL PLATES WITH MICROPOLAR FLUIDS

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ABSTRACT

The present analysis deals with the squeeze film lubrication of micropolar fluid between porous elliptical plates. Analytical solution of characteristics of the bearing such as squeeze film pressure, load and squeezing time presented and are also expressed graphically against various emerging physical quantities. Numerical results show that the bearing characteristics in the presence of porous medium are found to be decreased.

Keyword head: Porous, Elliptical plates, Micropolar fluids


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1. INTRODUCTION

Micropolar fluids are fluids which exhibit certain microscopic effects arising from the micro rotations of the fluid elements. The fluids with constant viscosity are known as Newtonian fluids. Lubricant is a substance that reduces friction and wear, providing smooth running and satisfactory life for machine elements. Lubricants are both liquids and solids such as mineral oils, silicon fluids, water, dry bearings, greases, gases etc. Many investigators have studied the different bearing system using micropolar fluid. Khonsari [1] studied that performance of finite length journal bearing lubricated with micropolar fluids. He has analysed that the frictional force is greater in micropolar fluid and friction coefficient is lesser compared to Newtonian fluid. Agrawal et al. [2] studied the squeeze film characteristics of externally pressurised bearings lubricated with micropolar fluids and found that the time of approach is more for the micropolar fluids as compared to the Newtonian fluids. Several investigators have studied the micropolar fluid with different bearing system [3-5]. They revealed that the importance of micropolar fluids on the performance of bearing characteristics such as load carrying capacity and time of approach...
for squeeze film bearings. Naduvannamani and Huggi [6] have carried out an in depth study on micropolar fluid and its impact on porous journal bearings. They have found that the effect of micropolar fluid enhance the load carrying capacity and correspondingly increases the squeeze film time. Naduvannamani and Kashinath [7] studied static and dynamic characteristics of short journal bearing with micropolar fluids. The authors discussed two different types of loads: a constant applied load and an alternative applied load. Further, it is observed that, the presence of microstructure additives and the negatively skewed surface roughness on the bearing surface increases the load carrying capacity and decreases the journal centre velocity due to reduction in the bearing under cyclic load. Naduvannamani et al. [8] studied the squeeze film lubrication of short porous partial journal bearing with micropolar fluids. They have found that the transverse surface roughness pattern improving the squeeze film characteristics in presence of one dimensional longitudinal surface roughness. Naduvannamani and Siddangouda [9] have analysed the effects of surface roughness with couple stresses on squeeze film lubrication with porous circular stepped plates and found that squeeze film characteristics increase (decrease) with increasing values of azimuthal (radial) roughness pattern. Naduvannamani and Siddangouda [10] analyzed the porous Rayleigh step bearings lubricated with couple-stress fluids. The load carrying capacity increases and coefficient of friction decreases for increasing the couplestress effects as compared to the corresponding Newtonian case. This theory suggests that, adverse effects of the presence of porous facing on the stator could be compensated with proper selection of lubricants with proper additives. The behaviour of micropolar fluids have been studied for different bearing configurations [11, 12]. They have analysed that use of micropolar lubricants in bearings results maximum load capacity than with a Newtonian lubricant.

Recently Roopa et al. [13] Studied the squeeze film characteristics of elliptical plates lubricated with micropolar fluid. So far, the author is not aware of the characteristics of a porous elliptical plates lubricated with micropolar fluid, hence in this paper we made an attempt to analyse the squeeze film characteristics of porous elliptical plates with micropolar fluid.

2. MATHEMATICAL FORMULATION
The schematic representation is as shown in Figure 1. The squeeze film between two elliptical plates, the upper plate is approaching to the fixed lower plate with the normal velocity \(v\). The upper surface of the plate is solid facing and the lower surface of the plate has surface of porous medium. The two plates are separated by a film thickness \(h\). \(a\) and \(b\) are semi major and semi minor axis of the elliptical plates.

![Figure 1 Geometry of Porous Elliptical Plates](image-url)
Under usual assumptions of the lubrication theory for thin films, the basic equations governing the flow of micropolar fluids are

\[
\left( \mu + \frac{\chi}{2} \right) \frac{\partial^2 u}{\partial y^2} + \chi \frac{\partial v_3}{\partial y} = \frac{\partial p}{\partial x} \quad (1)
\]

\[
\frac{\partial p}{\partial y} = 0
\]

\[
\left( \mu + \frac{\chi}{2} \right) \frac{\partial^2 w}{\partial y^2} - \chi \frac{\partial v_1}{\partial y} = \frac{\partial p}{\partial z} \quad (3)
\]

\[
\gamma \frac{\partial^2 v_3}{\partial y^2} - \chi \frac{\partial u}{\partial y} - 2\chi v_3 = 0 \quad (4)
\]

\[
\gamma \frac{\partial^2 v_1}{\partial y^2} + \chi \frac{\partial w}{\partial y} - 2\chi v_1 = 0 \quad (5)
\]

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (6)
\]

Where \((u, v, w)\) are the velocity components of the lubricant along \(x, y, z\)-directions and \((v_1, v_2, v_3)\) are micro rotational components, \(\chi\) and \(\gamma\) are the additional viscosity coefficients for micropolar fluids and \(\mu\) is the Newtonian viscosity coefficient.

The relevant boundary conditions are as follows:

At the upper elliptical plate surface \(y = h\)

\[
u_1 = v_3 = 0
\]

At the lower elliptical porous surface \(y = 0\)

\[
u_1 = v_3 = 0
\]

The modified Darcy equations are given by

\[
u^* = \frac{-k}{(\mu + \chi)} \frac{\partial p^*}{\partial x}, \quad w^* = \frac{-k}{(\mu + \chi)} \frac{\partial p^*}{\partial y}, \quad w^* = \frac{-k}{(\mu + \chi)} \frac{\partial p^*}{\partial z}
\]

Where \(k\) is the permeability of the porous medium and \(p^*\) is the pressure in the porous surface. Solving the equations \((1) - (5)\) subject to the boundary conditions \((7a), (7b), (8a)\) and \((8b)\) one can derive the expression for velocity components as:

\[
u = \frac{f_1 \partial p}{\mu \partial x} + \frac{D_1 f_2}{(1 - N^2)}
\]
\[ w = f_1 \frac{\partial p}{\partial z} + \frac{D_2 f_2}{(1 - N^2)} \]  \hspace{1cm} (10)

Where \n
\[ f_1 = \frac{y^2}{2} - \frac{N^2 h (\text{Cosh}m^2 - 1)}{m \text{Sinh}m^2}, \quad f_2 = y - \frac{N^2}{m} \left\{ \text{Sinh}m^2 - (\text{Cosh}m^2 - 1) \frac{\text{Cosh}m^2 - 1}{\text{Sinh}m^2} \right\} \]

\[ D_1 = -\frac{(1 - N^2)}{2} \left\{ \frac{h}{2\mu} \frac{\partial p}{\partial x} \right\}, \quad D_2 = -\frac{(1 - N^2)}{2} \left\{ \frac{h}{2\mu} \frac{\partial P}{\partial x} \right\}, \quad m = \frac{N}{L} \]

Integrating the continuity equation (6) w. r. t. \( y \) over the film thickness and by substituting the expressions for \( u \) and \( w \) from the equation (8) and (9), the modified Reynold's equation obtained as:

\[ \frac{\partial^2 p}{\partial x^2} + \frac{\partial^3 p}{\partial z^2} = \frac{12\mu dh/dt}{f(N, L, h) + \frac{12\mu k\delta}{(\mu + \chi)}} \]  \hspace{1cm} (11)

Where \n
\[ f(N, L, h) = h^3 + 12L^2 h - 6NLh^2 \text{coth} \left( \frac{Nh}{2L} \right), \quad L = \left( \frac{\chi}{4\mu} \right)^{1/2}, \quad N = \left( \frac{\chi}{\chi + 2\mu} \right)^{1/2} \]

\( N \) is called the coupling number which characterizes the coupling between the Newtonian and microrotational viscosities. \( L \) represents the interaction between the micropolar fluid and the film gap and is termed as the characteristics length of the micropolar fluid.

The boundary conditions of pressure distribution for elliptical plates are as follows:

\[ p(x_i, z_i) = 0 \]  \hspace{1cm} (12)

\[ \frac{x_i^2}{a^2} + \frac{z_i^2}{b^2} = 1 \]  \hspace{1cm} (13)

The expression for pressure distribution can be derived from the equation (11) subject to the conditions (12) as:

\[ p = -\frac{12\mu dh/dt}{f(N, L, h) + \frac{12\mu k\delta}{(\mu + \chi)}} \left\{ \frac{a^2b^2}{2(a^2 + b^2)} \left( 1 - \frac{x_i^2}{a^2} - \frac{z_i^2}{b^2} \right) \right\} \]  \hspace{1cm} (14)

The following dimensionless variables are introduced:

\[ h^* = \frac{h}{h_0}, \quad L^* = \frac{L}{h_0}, \quad a^* = \frac{a}{b}, \quad x_i^* = \frac{x_i}{a}, \quad z_i^* = \frac{z_i}{b}, \quad \psi = \frac{k\delta}{h_0}, \quad p^* = -\frac{ph_0^3}{\mu dh/dt ab} \]

Using the above dimensionless scheme pressure distribution in dimensionless form can be written as:
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\[ p^* = \left( \frac{a^*}{a^{*2} + 1} \right) \frac{6(1 - x^{*2} - z^{*2})}{F(N, L^*, h^*) + \frac{12\psi(1 - N^*2)}{(1 + N^2)}} \]  

\[ F(N, L^*, h^*) = h^*3 + 12L^2h^* - 6NL^*h^2 \coth \left( \frac{Nh^*}{2L} \right) \]  

(15)

Where

The load carrying capacity of the porous elliptical plates is evaluated by integrating the fluid film pressure acting on the plate as:

\[ W = \int_{a}^{b} \int_{-a}^{b} p dz \, dx \]

\[ = -\frac{12\mu dh/\mu}{f(N, L, h) + \frac{12\mu k \delta}{(a^2 + b^2)} \left( \frac{a^*b^2}{(a^2 + b^2)} \right) ab\pi} \]

(16)

The load carrying capacity in dimensionless form is given by

\[ W^* = -\frac{W h_{0}^3}{\mu (dh/dt) a^2 b^2} = \left( \frac{a^*}{a^{*2} + 1} \right) \frac{3}{F(N, L^*, h^*) + \frac{12\psi(1 - N^*2)}{(1 + N^2)}} \]

(17)

The squeeze film time can be derived by integrating the equation (17) for the film thickness to decrease from \( h_{0} \) at \( t = t_{0} \) to \( h_{1} \) at \( t = t_{1} \) as:

\[ t = -\left( \frac{a^*b^2}{(a^2 + b^2)} \right) \frac{ab\pi}{4W} \int_{h_{0}}^{h_{1}} \frac{12\mu dh}{f(N, L, h) + \frac{12\mu k \delta}{(a^2 + b^2)}} \]

(18)

In dimensionless form is given by

\[ T^* = -\int_{1}^{\frac{h_{0}}{h_{1}}} \frac{W h_{0}^3 dt}{\mu a^2 b^2} = \left( \frac{3a^*}{a^{*2} + 1} \right) \int_{1}^{h_{1}} \frac{dh^*}{F(N, L^*, h^*) + \frac{12\psi(1 - N^*2)}{(1 + N^2)}} \]

(19)

3. RESULTS AND DISCUSSIONS

The characteristics of porous elliptical bearing lubricated with micropolar fluids is presented for the parameters \( L^* \) \( = L/h_{0} \) \( 1/2 \) where \( L = (\gamma/4\mu)^{1/2} \), the parameter \( L^* \) characterizes the interaction of the bearing geometry with the lubricant properties, and \( N \{= \chi/(\chi + 2\mu) \}^{1/2} \), the permeability parameter \( \psi(= k\delta/h_{0}^3) \) and the aspect ratio \( a^* \). The effect of permeability is analysed through the dimensionless parameter \( \psi \). As the permeability parameter \( \psi = 0 \) the squeeze film characteristics analysed in this paper reduce to the non-porous case studied by Roopa et.al [15] and are presented in Table I. Excellent agreements were found.
3.1. Squeeze film Pressure

Variation of non-dimensional squeeze film pressure $P^*$ with coupling number $L^*$ and permeability parameter $\psi$ as a function of $x^*$ is illustrated in Figure 2 with $h = 0.5, N = 0.2, a^* = 1$ and $z = 0$. It is interesting to note that with the increasing the values of $\psi$, the pressure $P^*$ and also $P^*$ increases for the larger values of $L^*$.

Figure 3 displays the variation of $P^*$ with permeability parameter $\psi$ and coupling number $N$ as a function of coordinate $x^*$ with $h = 0.5, L = 0.2, a^* = 1$ and $z = 0$. It is observed that the increasing values of $\psi$ show a significant decrease in $P^*$. Further it is noted that the increasing value of $N$ is to increase the pressure $P^*$ for the both porous and non-porous cases. Figure 4 represents the film pressure $P^*$ versus $x^*$ and $a^*$ (aspect ratio) with different parametric values $h = 0.5, N = 0.2, L = 0.2, a^* = 1$ and $z = 0$. The impact of aspect ratio on the fluid film pressure distribution is clearly seen. It is observed that the permeability effect is to decrease the pressure $P^*$ significantly for smaller values of $a^*$. Further, it is also observed that the non-dimensional pressure, decreases for increasing values of the permeability parameter $\psi$. The reason is that the larger permeability values means there are more voids available in the porous facing, permitting the quick escape of the lubricant.
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Figure 3: Variation of nondimensional pressure $P^*$ with $x^*$ for different values of $\psi$ and $N$ with $h = 0.5, L^* = 0.2, a^* = 1$ and $z = 0$.

Figure 4: Variation of nondimensional pressure $P^*$ with $x^*$ for different values of $\psi$ and $a^*$ with $h = 0.5, N = 0.2, L^* = 0.2$ and $z = 0$.

3.2. Load carrying capacity

The variation of non-dimensional load carrying capacity $W^*$ with $\log_{10}(a^*)$ for various values of $\psi$ and $L^*$ at $N = 0.5$ and $h^* = 0.5$ is depicted in the figure 5. Here the dotted lines represent the non-porous case and solid lines represent the porous case. It is observed that $W^*$ decreases for increasing values $\psi$ as compared to the corresponding porous case. From the figure it is also clear that, $W^*$ increases for increasing values of $L^*$. 

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Figure 6 shows the variation of non-dimensional load carrying capacity $W^*$ with $\log_{10}(a^*)$ for various values of $\psi$ and $N$ at $L^* = 0.2$ and $h^* = 0.5$. As illustrated in the figure, the effect of $\psi$ decreases $W^*$ as compared to non-porous case. It is seen that $W^*$ increases for increasing values of $N$. Figure 7 describes the variation of non-dimensional load carrying capacity $W^*$ with $\log_{10}(a^*)$ for various values of $\psi$ and $h^*$ at $N = 0.2$ and $L^* = 0.2$. It is observed that the increasing values of $\psi$ decrease the value of $W^*$. Further, it is observed that $W^*$ increase for decreasing values of $h^*$.
3.3. Squeeze film Time

Figure 8 illustrates the variation of squeeze film time $T^*$ for different values of parameter $\psi$ and $L^*$ as a function of $h^*$ with $N = 0.2, a^* = 1$ and $z = 0$. It is observed that the effect permeability is to decrease the squeezing time for each given film height as compared to solid case. Moreover, the effects of permeability $\psi$ increases results in further decreasing the squeezing time. When the values $\psi$ increases, the fluid easily percolates into the porous region as a result load capacity
decrease and thus time leads to attain a prescribe film height decreases. This figure shows that an increase of the parameter $L$ increases the squeezing time.

Figure 9 and 10 illustrate the variation in squeezing with permeability parameter $\psi$ as a function of $h^*$ for different values of $N$ (with and $\psi$ with $L^* = 0.2$ and $a^* = 1$) and aspect ratio $a^*$ (with $L^* = 0.2$ and $N^* = 0.2$) respectively. The dimensionless squeeze film time decreases with increasing $h^*$ and the effect of permeability $\psi$. These figures indicate that the squeezing time decreases with increasing the permeability effects while squeezing time increase an increase of the parameter $N$ and the aspect ratio $a^*$. It is interesting to note that the effect of the permeability parameter is to decrease the squeezing time as compared to non-porous case.

![Graph showing variation of nondimensional time $T^*$ with $h^*$ for different values of $\psi$ and $L^*$ with $N = 0.2$, $a^* = 1$ and $z = 0$.](image)

**4. CONCLUSIONS**

The theoretical study on the characteristic of porous elliptical plates lubricated with micropolar fluid is presented in this paper. As $\psi \to 0$ the squeeze film characteristics obtained in this study reduce that of smooth case studied by Roopa et.al.[15]. According to the results discussed, the following are the main salient predictions:

- The presence of porous facing on the elliptical plate decrease the squeeze film pressure, load-carrying capacity and squeezing time.
- The squeeze film pressure, load-carrying capacity and squeezing time increase with the increasing the values of $L^*, N$ and $a^*$. 

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Figure 9: Variation of nondimensional time $T^*$ with $h^*$ for different values of $\psi$ and $N$ with $L^* = 0.2$ and $a^* = 1$.

Figure 10: Variation of nondimensional time $T^*$ with $h^*$ for different values of $\psi$ and $a^*$ with $L^* = 0.2$ and $N = 0.2$.
Table I: Variation of dimensionless pressure $P^*$, load-carrying capacity $W^*$ and squeezing time $T^*$ at $h^* = 0.5, a^* = 1.5, x^* = 0.6$ and $z = 0$.

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<th>Roopa et.al [15]</th>
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<tr>
<td>$W^*$</td>
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