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AVERAGE RUN LENGTH FOR EXPONENTIATED DISTRIBUTION UNDER TRUNCATED LIFE TEST

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ABSTRACT

Statistical quality control discusses about the screening and preserving the quality of products and services. Control charts are used widely in manufacturing industry. The acceptance sampling plans for truncated life tests are regularly practiced to regulate the sample size from a lot under inspection. Traditional control chart needs more samples for testing the quality of the product. In this paper, failure time of a product follows non-normal distributions like Exponentiated Gamma Distribution, Exponentiated Lomax Distribution and Beta Weibull Distribution. The defectives that are found are put into truncated life test and upper and lower control limits are computed. Control charts using these distributions are designed to monitor the mean shift by detecting the number of unsuccessful products at a definite time. The probability for the in-control process and out of control process are estimated from the sample data. The Average Run Length (ARL) allows an assessment to be considered for various screening policies and based on the various mean shift, the ARL values are computed for various distributions and compared.

Key words: Control charts, Exponentiated Gamma Distribution, Exponentiated Lomax Distribution, Average Run Length.

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1. INTRODUCTION

Acceptance sampling plan is a crucial device in the Statistical Quality Control. It is not likely to accomplish hundred percent inspection owing to many reasons in statistical quality control experiment. Control charts are powerful techniques in determining whether a process is in control or out of control. Attribute and variable control charts are two partitions of control charts. Attribute data is used designing attribute control chart and variable data is used for designing variable control chart. Various control charts have been developed by the researchers in recent years[1,2,3,4,5,6,7,8,9,10]. Manufacturing industries involves many complex process by maintaining the output parameter according to the input specifications. But there are uncontrollable measures that degrade the performance of the entire process.

In this paper, the automobile specifications data are considered from UCI repository which has 26 attributes. The average number of defectives are computed based on the number of defectives found which is used to compute the values of UC UCL and LCL. The number of defectives is tested to be within control limits specifications. To compute the number of defectives, few attributes are scaled different from the UCI repository.

Data set used in this paper consist of 200 observations for time in months until next service for a certain automobile specification. It is considered that the product has a failure time and undergoes one of the following four distributions.

- Exponentiated Gamma Distribution
- Exponentiated Lomax Distribution
- Beta Weibull Distribution
- Log-Logistic Distribution

These distributions is most commonly used in modelling survival processes with various shape parameters and scale parameters in hazard function. The use of exponentiated gamma, exponentiated lomax, beta Weibull and log-logistic fits well for real time applications.

2. CONTROL CHARTS FOR EXPONENTIATED GAMMA DISTRIBUTION

If the failure time of a product follows an Exponentiated Gamma distribution then its CDF is given by

$$F(t;\theta,\lambda) = [1 - e^{-\lambda x}(tx + 1)]^{\theta} \quad [1]$$

where λ is the scale parameter and θ is the shape parameter. The product whose mean life has an exponentiated Gamma distribution is given by

$$\mu = \frac{\theta}{\lambda}[2 + A_1(\theta)] \quad [2]$$

where $A_1(\theta) = \sum_{j=1}^{\infty}\sum_{k=0}^{j}(-1)^j \binom{\theta-1}{j}\binom{j}{k}\frac{\Gamma(1+k+2)}{(1+j)^{1+k+2}}$

At a specified time t_o, the number of failed product is noted in order to monitor the mean shift for designing control chart. At the time t_o the failed item has a probability given by

$$P = [1 - e^{-\lambda t_o}(\lambda t_o + 1)]^{\theta} \quad [3]$$

If the truncation time t_o is considered as the multiple of the in-control process mean $t_o = a\mu_o$ at some constant time a, called truncated constant then the scale parameter λ can be expressed in form of μ in (1) and Eq.(3) is given by

$$p = \left\{1 - \exp\left(-\frac{\theta}{\mu}\left[2 + \sum_{j=1}^{\infty}\sum_{k=0}^{j}(-1)^j \binom{\theta-1}{j}\binom{j}{k}\frac{\Gamma(1+k+2)}{(1+j)^{1+k+2}}\right)a\mu_o\right)\right][\left(\frac{\theta}{\mu}(2 + \sum_{j=1}^{\infty}\sum_{k=0}^{j}(-1)^j \binom{\theta-1}{j}\binom{j}{k}\frac{\Gamma(1+k+2)}{(1+j)^{1+k+2}}\right)a\mu_o) + 1]\Big]\Big\}^{\theta} \quad [4]$$

For the in control process ie.$\mu = \mu_o$,then Eq.[4] becomes

$$p_0 = \left\{1 - \exp(-\theta a\left[2 + \sum_{j=1}^{\infty}\sum_{k=0}^{j}(-1)^j \binom{\theta-1}{j}\binom{j}{k}\frac{\Gamma(1+k+2)}{(1+j)^{1+k+2}}]\right) [(\theta a\left(2 + \sum_{j=1}^{\infty}\sum_{k=0}^{j}(-1)^j \binom{\theta-1}{j}\binom{j}{k}\frac{\Gamma(1+k+2)}{(1+j)^{1+k+2}})a\mu_o\right) + 1]\right]\right\}^{\theta} \quad [5]$$

The control limits are

$$UCL = np_0 + k\sqrt{np_0(1-p_0)} \quad [6]$$

$$LCL = \max[0, np_0 - k\sqrt{np_0(1-p_0)} \quad [7]$$

where k is called the coefficient of the control limits.

From the automobile specification data, 20 random samples are taken for each subgroup containing 200 observations .The failure data is generated by changing the range of the wheel-base from 91.2 to 112.5 and length from 170 to 191. It is found that $\bar{p}$ =0.35 (where $\bar{p}$ is the average probability of the failed items) from the data and then $\bar{D}$=n$\bar{p}$ =7.10. From table 1 it is noticed that a=0.827, k=2.6995 and hence the control limits are UCL=12.87 and LCL= 1.32.

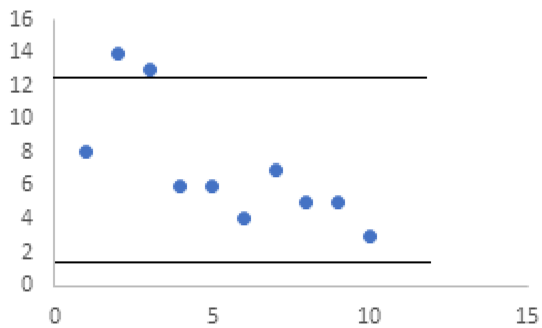


Figure 1 Proposed data points and the control limits

The probability p_o for the in-control process is estimated from the preliminary sample data. The probability that the process is said to be in control when it is actually in control is given by

$$P_{in}{}^{o} = \sum_{d=LCL+1}^{UCL}\binom{n}{d} p_o{}^{d}(1-p_o)^{n-d} \quad [8]$$

Let us consider that the mean of the process is shifted from μ_o to μ_1. Then the Eq. [4] becomes

$$p_1 = \left\{1 - \exp(-\frac{\theta}{\mu_1}\left[2 + \sum_{j=1}^{\infty}\sum_{k=0}^{j}(-1)^j \binom{\theta-1}{j}\binom{j}{k}\frac{\Gamma(1+k+2)}{(1+j)^{1+k+2}})a\mu_o)\right] [(\frac{\theta}{\mu_1}(2 + \sum_{j=1}^{\infty}\sum_{k=0}^{j}(-1)^j \binom{\theta-1}{j}\binom{j}{k}\frac{\Gamma(1+k+2)}{(1+j)^{1+k+2}})a\mu_o) + 1]]\right\}^{\theta} \quad [9]$$

If μ_1=fμ_o is the mean shift, then [9] becomes

$$p_1 = \left\{1 - \exp(-(\theta a)/f\left[2 + \sum_{j=1}^{\infty}\sum_{k=0}^{j}(-1)^j \binom{\theta-1}{j}\binom{j}{k}\frac{\Gamma(1+k+2)}{(1+j)^{1+k+2}}]\right) (\frac{\theta}{f}(2 + \sum_{j=1}^{\infty}\sum_{k=0}^{j}(-1)^j \binom{\theta-1}{j}\binom{j}{k}\frac{\Gamma(1+k+2)}{(1+j)^{1+k+2}})a) + 1]]\right\}^{\theta} \quad [10]$$

Since the mean has shifted to μ_1 , the in-control probability is given by

$$P_{in}{}^{1} = \sum_{d=LCL+1}^{UCL}\binom{n}{d} p_1{}^{d}(1-p_1)^{n-d} \quad [11]$$

The ARL of the in-control process is

$$ARL_O = \frac{1}{1-P_{in}{}^{O}} \quad [12]$$

and the out-of-control ARL is given by

$ARL_1 = \frac{1}{1-P_{in}{}^1}$ [13]

For a given n, the truncated time constant a and the values of the control constant k are computed such that the values of in-control ARL (ARL_O) and specified ARL, say r_o are very close to each other. Then, ARL_1 is calculated for different values of the mean shift parameter *f*.

Table 1 Values of ARL for the control chart using Exponentiated Gamma for n=20

Shift	a=0.827,k=2.6995,θ =2,r_o=113	a=0.87,k=2.85368, θ=2,r_o=187
1	113.94	187.06
0.9	28.17	37.54
0.8	6.61	7.90
0.7	2.17	2.39
0.6	1.18	1.22
0.5	1.00	1.01
0.4	1.00	1.00
0.3	1.00	1.00
0.2	1.00	1.00
0.1	1.00	1.00

For specified parameters, the above values of ARL is generated. The parameters can be changed depending on the user's need.

3. DESIGN OF CONTROL CHARTS FOR EXPONENTIATED LOMAX DISTRIBUTION

If the product whose failure time follows an Exponentiated Lomax distribution then its CDF is given by

$$F(x) = \left[1-(1+\lambda x)^{-\theta}\right]^{\alpha} \quad [14]$$

where λ is the scale parameter ,θ is the shape parameter and α is the second shape parameter. The mean life of a product for the exponentiated Lomax distribution is given by

$$\mu = \frac{\alpha}{\lambda}\left[\beta\left(1-\frac{1}{\theta},\alpha\right)-\beta(1,\alpha)\right] \quad [15]$$

At a specified time t_o, the number of failed product is noted in order to monitor the mean shift for designing control chart. The probability that an item fails by time t_o is given by

$$p = \left[1-(1+\lambda t_0)^{-\theta}\right]^{\alpha} \quad [16]$$

If the truncation time t_o is considered as the multiple of the in-control process mean t_o =aμ_o at some constant time a, called truncated constant then the scale parameter λ can be expressed in terms of μ in Eq.[14] and Eq.[16] is given as

$$p=[1-(1+\frac{\alpha}{\mu}\,[\beta(1-\frac{1}{\theta},\alpha)-\beta(1,\alpha)]\,a\mu_o)^{-\theta}]^{\alpha} \quad [17]$$

By following the same algorithm as in exponentiated Gamma distribution we arrive at

$$p_0=[1-(1+a\alpha\,[\beta(1-\frac{1}{\theta},\alpha)-\beta(1,\alpha)]\,)^{-\theta}]^{\alpha} \quad [18]$$

$$p_1=[1-(1+(\frac{a}{f})\alpha\,[\beta(1-\frac{1}{\theta},\alpha)-\beta(1,\alpha)]\,)^{-\theta}]^{\alpha} \quad [19]$$

Table 2 is found from Eq.[6] and [7] for the control limits(UCL and LCL) , Eq.[8] and [11] for $p_{in}{}^{o}$ and $p_{in}{}^{1}$, Eq.[12] for $\mathrm{AR}L_O$ and Eq.[13] for $\mathrm{AR}L_1$.

For the same 200 observations, 20 samples are taken for each subgroup. The failure data is generated by changing only the range of the wheel-base from91.2 to 112.5 It is found that $\bar{p}$ =0.38 (where $\bar{p}$ is the average probability of the failed items) from the data and then $\bar{D}$=n$\bar{p}$ =7.7. From the Table 2, the values of a=0.827 and k=2.6995 is noted and hence the control limits are calculated as UCL=13.58 and LCL=1.83. Using UCL and LCL, the values of ARL are obtained using the formula in [12] and [13].

TABLE 2 Values of ARL for the control chart using Exponentiated Lomax when n=20

	θ =2,α=4	θ=2, α=3
shift	a=0.827,k=2.6995,r_o=139.26	a=0.716,k=2.87988,r_o=539
1	139.26	539.98
0.9	107.30	382.37
0.8	46.36	140.52
0.7	19.06	49.04
0.6	8.40	18.14
0.5	4.09	7.33
0.3	2.26	3.35
0.3	1.45	1.81
0.2	1.11	1.20
0.1	1.01	1.01

4. DESIGN OF CONTROL CHARTS FOR BETA WEIBULL DISTRIBUTION

If the failure time of a product follows a Beta Weibull distribution, then its cumulative distribution function is given by

$$F(t)=\frac{1}{B(m,n)}\sum_{j=0}^{m-1}\binom{m-1}{j}(-1)^{j}\frac{\{1-e^{-(n+j)(\lambda t)^{c}}\}}{(n+j)} \quad [20]$$

Where λ is the scale parameter and c is the shape parameter. The mean life of a product for the beta weibull distribution is given by

$$\mu=\frac{\Gamma(\frac{1}{c+1})}{\lambda B(m,n)}\sum_{j=0}^{m-1}\binom{m-1}{j}\frac{(-1)^{j}}{(n+j)^{\frac{1}{(c+1)}}} \quad [21]$$

At a specified time t_o, the number of failed product is noted in order to monitor the mean shift for designing control chart. The probability that an item fails by time t_o is given by

$$p = \frac{1}{B(m,n)} \sum_{j=0}^{m-1} \binom{m-1}{j} (-1)^j \frac{\left\{1-e^{-(n+j)(\lambda t_o)^c}\right\}}{(n+j)} \quad [22]$$

If the truncation time t_o is considered as some constant time of the process mean which is in control then t_o =aμ_o . The scale parameter λ is expressed in terms of μ in Eq.[20] then we write Eq.[22] as

$$p = \frac{1}{\beta(m,n)} \sum_{j=0}^{m-1} \binom{m-1}{j} (-1)^j \left(\frac{1}{n+j}\right) \left\{1 - exp\left(-(n+j)\left(\frac{\Gamma\left(\frac{1}{c+1}\right)}{\mu\beta(m,n)} \sum_{j=0}^{m-1} \binom{m-1}{j} \frac{(-1)^j a\mu_o}{(n+j)^{\frac{1}{c+1}}}\right)^c\right)\right\} \quad [23]$$

By following the same algorithm as in exponentiated Gamma distribution we arrive at

$$p_0 = \frac{1}{\beta(m,n)} \sum_{j=0}^{m-1} \binom{m-1}{j} (-1)^j \left(\frac{1}{n+j}\right) \left\{1 - exp\left(-(n+j)\left(\frac{\Gamma\left(\frac{1}{c+1}\right)}{\mu\beta(m,n)} \sum_{j=0}^{m-1} \binom{m-1}{j} \frac{(-1)^j a}{(n+j)^{\frac{1}{c+1}}}\right)^c\right)\right\} \quad [24]$$

$$p_1 = \frac{1}{f\beta(m,n)} \sum_{j=0}^{m-1} \binom{m-1}{j} (-1)^j \left(\frac{1}{n+j}\right) \left\{1 - exp\left(-(n+j)\left(\frac{\Gamma\left(\frac{1}{c+1}\right)}{\mu\beta(m,n)} \sum_{j=0}^{m-1} \binom{m-1}{j} \frac{(-1)^j a}{(n+j)^{\frac{1}{c+1}}}\right)^c\right)\right\} \quad [25]$$

Table 3 is compute with Eq.[6] and [7] for the control limits(UCL and LCL) , Eq.[8] for $p_{in}{}^o$, Eq.[9] as $p_{in}{}^1$, Eq.[12] for ARL_O and Eq.[13] for ARL_1.

For the same 200 observations, 20 samples are taken for each subgroup. The failure data is generated by changing only the range of the wheel-base from 91.2 to 112.5. It is found that $\overline{p}$ =0.38 (where $\overline{p}$ is the average probability of the failed items) from the data and then $\overline{D}$=n$\overline{p}$ =6.8. From Table 3 a=0.827,k=2.6995 and hence the control limits are UCL=11 and LCL=2. Using UCL and LCL, the ARL is obtained as shown in Table 3 using the formula [12] and [13].

Table 3 Values of ARL for the control chart using Beta Weibull for n=20 for c=1

Shift	a=0.827,k=2.6995,m=1,n= 1,r_o=256	a=0.87,k=2.85368, m=2,n=1,r_o=533
1	256.32	533.63
0.9	79.35	148.73
0.8	23.69	39.40
0.7	7.65	11.23
0.6	2.91	3.75
0.5	1.46	1.65
0.4	1.05	1.09
0.3	1.00	1.00
0.2	1.00	1.00
0.1	1.00	1.00

5. DESIGN OF CONTROL CHARTS FOR LOG LOGISTIC DISTRIBUTION

If the product whose failure time follows a log logistic distribution, then its cumulative distribution function is given by

$$F(x,\alpha,\beta) = \frac{1}{1+(\frac{x}{\alpha})^{-\beta}} \qquad [26]$$

Where α and β are the scale and the shape parameter respectively. The mean life of a product for the log logistic distribution is given by

$$\mu = \frac{\frac{\alpha\pi}{\beta}}{\sin(\frac{\pi}{\beta})} \qquad [27]$$

At a specified time t_o, the number of failed product is noted in order to monitor the mean shift for designing control chart. The probability that an item fails by time t_o is given by

$$p = \frac{1}{1+\left(\frac{t_o}{\alpha}\right)^{-\beta}} \qquad [28]$$

If the truncation time t_o is considered as some constant time of the process mean which is in control then t_o =aμ_o . The scale parameter λ is expressed in terms of μ in Eq.[26] then we write Eq.[28] as

$$p = \frac{1}{1+(\frac{a\mu_o}{\frac{\beta}{\pi}\mu\sin\left(\frac{\pi}{\beta}\right)})^{-\beta}} \qquad [29]$$

By following the same algorithm as in exponentiated Gamma distribution we arrive at

$$p_o = \frac{1}{1+(\frac{a}{\frac{\beta}{\pi}\sin(\frac{\pi}{\beta})})^{-\beta}} \qquad [30]$$

$$p_1 = \frac{1}{1+(\frac{a}{\frac{\beta}{\pi}f\sin(\frac{\pi}{\beta})})^{-\beta}} \qquad [31]$$

Table 3 is compute with Eq.[6] and [7] for the control limits(UCL and LCL) , Eq.[8] for ${p_{in}}^o$, Eq.[9] as ${p_{in}}^1$, Eq.[12] for ARL_O and Eq.[13] for ARL_1.

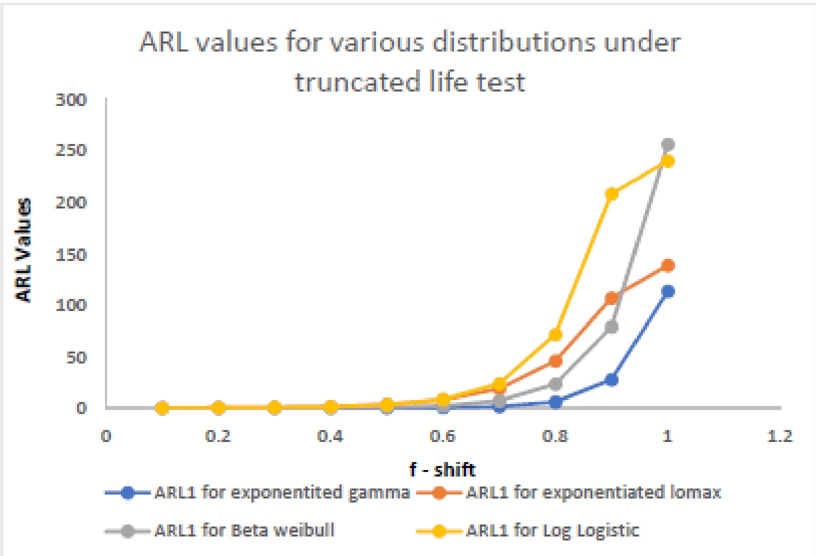
Table 1 Values of ARL for the control chart using log logistic for n=20

	$\beta = 2$	$\beta = 3$
shift	a=0.827,k=2.6995,r_o=240.2378	a=0.87,k=2.85368,r_o= 354.75
1	240.23	354.75
0.9	208.57	183.98
0.8	72.14	31.55
0.7	24.14	7.49
0.6	9.10	2.67
0.5	4.00	1.44
0.4	2.11	1.09
0.3	1.36	1.01
0.2	1.08	1.00
0.1	1.00	1.00

Various distributions like exponentiated Lindey, inverse gamma and inverse Weibull distributions were also carried out. The values of average run length were computed and found to be too large or negative. Hence the values were not tabulated.

Table 4 Comparison table for ARL values

	a=0.827,k=2.6995			
Shift	Exponentiated Gamma	Exponentiated Lomax	Beta Weibull	Log logistic
1	113.94	139.26	256.32	240.23
0.9	28.17	107.30	79.35	208.52
0.8	6.61	49.36	23.69	72.14
0.7	2.17	19.06	7.65	24.14
0.6	1.18	8.40	2.91	9.10
0.5	1.00	4.09	1.46	4.00
0.4	1.00	2.26	1.05	2.11
0.3	1.00	1.45	1.00	1.36
0.2	1.00	1.11	1.00	1.08
0.1	1.00	1.01	1.00	1.00



6. CONCLUSION

The ARLvalues for various shift is found to be less in Exponentiated Gamma than Exponentiated Lomax and Beta Weibull. The values of ARL is found to be one, when the shift is 0.6. Exponentiated Gamma gives a better result than earlier existing papers [2]. It is also found that ARL for Beta Weibull is more when shift is one but decreasing drastically as shift decreases. Exponentiated Lomax is better than Beta Weibull when the shift is more than or equal to one. The values of ARL is more for log-logistic compared to other distributions. The control parameters can be considered depending on the practical use. The ARL values of Exponentiated Lindey, inverse gamma and inverse Weibull distributions was found to be negative or too large and cannot be used for real time application. Exponentiated Gamma is suitable and much flexible to monitor the lifetimes of quality products in manufacturing industry. For smaller shift Exponentiated Gamma works well and larger shift Exponentiated Lomax is better. Exponentiated Gamma is suitable and much flexible to monitor the lifetimes of quality products in manufacturing industry.

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