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DIVIDED SQUARE DIFFERENCE CORDIAL LABELING GRAPHS

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ABSTRACT

Let $G = (V, E)$ *be a simple graph and* $f: V \to \{1,2,3, \dots |V|\}$ *be bijection. For each edge* uv, *assign the label 1 if* $\left|\frac{(f(u))^2-(f(v))^2}{f(u)-f(v)}\right|$ *is odd and the label 0 otherwise.* f *is called Divided square difference Cordial Labeling if* $\left|e_f(0) - e_f(1)\right| \le 1$, *where* $e_f(1)$ *and* $e_f(0)$ *denote the number of edges labeled with 1 and not labeled with 1 respectively. In this paper, the concepts of Divided square difference cordial labeling behavior of path, cycle,* $K_{1,n}, K_{2,n}, P_n + K_1$ *and* $P_n + 2K_1$ *are introduced.*

Keywords: Divided square differencecordial labeling; $K_{1,n}$; $K_{2,n}$; $P_n + K_1$; $P_n + 2K_1$.

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1. INTRODUCTION AND PRELIMINARIES

In 1736. Euler first introduced the concept of graph theory. All graphs in this paper are finite, simple and undirected. We follow the basic notation and terminology of graph theory as in [7], while for number theory we refer to Burton [6] and of graph labeling as in [3]. Most graph labeling methods trace their origin to one introduced by Rosa [8] in 1967, or one given by Graham and Sloane [4] in 1980. The concept of cordial labeling was introduced by Cahit [1]. Dhavaseelan et.al [2] introduced the concept of even sum cordial labeling graphs. The concept of divisor cordial labeling was introduced by P. Lawrence Rozario Raj and R. Valli [5].

Let $G = (V, E)$ be a simple graph and $f: V \to \{1,2,3, \dots |V|\}$ be bijection. For each edge uv, assign the label 1 if $\left|\frac{(f(u))^2-(f(v))^2}{f(u)-f(v)}\right|$ is odd and the label 0 otherwise. f is called Divided square difference Cordial Labeling if $\left|e_f(0) - e_f(1)\right| \leq 1$, where $e_f(1)$ and $e_f(0)$ denote the number of edges labeled with 1 and not labeled with 1 respectively. In this paper, the concepts of Divided square difference cordial labeling behavior of path, cycle, $K_{1,n}, K_{2,n}, P_n + K_1$ and $P_n + 2K_1$ are introduced.

Definition 1.1

The Graph labeling is an assignment of numbers to the edges or vertices or both subject to certain condition(s). If the domain of the mapping is the set of vertices (edges), then the labeling is called a vertex (edge) labeling.

Graph labeling has large number of applications in Mathematics as well as in several areas of Computer science and Communication networks. For a dynamic survey on various graph labeling problems we refer to Gallian [3].

Definition 1.2

A mapping $f: V(G) \to \{0,1\}$ is called *binary vertex labeling* of G and $f(V)$ is called the label of the vertex v of G under f.

The concept of cordial labeling was introduced by Cahit [1].

Definition 1.3

A binary vertex labeling f of a graph G is called a C*ordial labeling* if $\left|v_f(0) - v_f(1)\right| \leq 1$ and $\left|e_f(0) - e_f(1)\right| \leq 1$.

A graph G is cordial if it admits cordial labeling.

Definition 1.4 [5]

Let $G = (\mathrm{V}(G), \mathrm{E}(G))$ be a simple graph and $f: \mathrm{V} \to \{1,2,\dots,|V(G)|\}$ be a bijection. For each edge uv, assign the label 1 if $f(u)/f(v)$ or $f(v)/f(u)$ and the label 0 otherwise. The function f is called a *Divisor cordial labeling* if $\left|e_f(0) - e_f(1)\right| \leq 1$. A graph with a divisor cordial labeling is called a divisor cordial graph.

2. MAIN RESULT

Definition 2.1

Let G be a Divided square difference cordial graph of odd size with the labeling f. If $e_f(0) = e_f(1) + 1$, then G is called a Divided square difference dominated cordial graph and if $e_f(1) = e_f(0) + 1$, then non-divided square difference dominated cordial graph.

Remark 2.2

We may change the Divided square difference dominated cordial graph into non Divided square difference dominated cordial graph by suitably adjusting the label and vice versa. It is showed in the following example.

Example 2.3

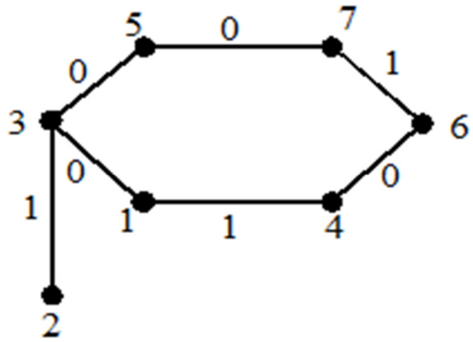

Figure 1

In the above graph, $e_f(0) = e_f(1) + 1$

By adjusting the label of the graph, we can change into non-Divided square difference dominated cordial graph which is given as follows.

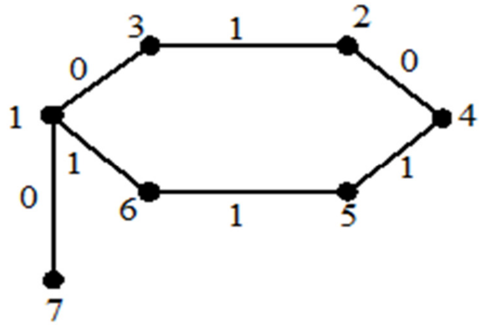

Figure 2

In the above graph, $e_f(1) = e_f(0) + 1$

Remark 2.4

We cannot always change from Divided square difference dominate cordial graph into non-divided square difference dominated cordial graph and vice versa.

Remark 2.5

Every Complete graph need not be Divided square difference cordial graph, when $n \geq 4$.

Proposition 2.6

If G is a Divided square difference cordial graph then $G - e$ is also a Divided square difference cordial graph.

Proof

Let G be a Divided square difference cordial graph of n vertices. Suppose n is odd, then we construct the graph having $e_f(0) = e_f(1) = \frac{n+1}{2}$. Let e be an edge in G which is labeled either 0 or 1.

Then $G - e$, we have either $e_f(0) = e_f(1) + 1$or $e_f(1) = e_f(0) + 1$ and hence $|e_f(0) - e_f(1)| \leq 1$.Thus the graph $G - e$ is a Divided square difference graph.

Similarly we can prove the proposition for when n is even.

Example 2.7

When $n = 8$

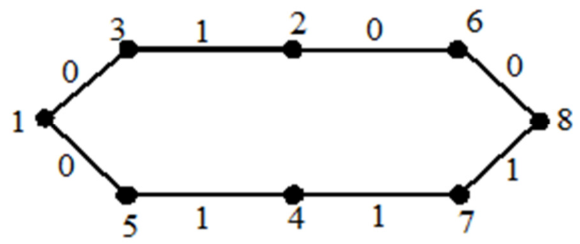

Figure 3

When $n = 7$

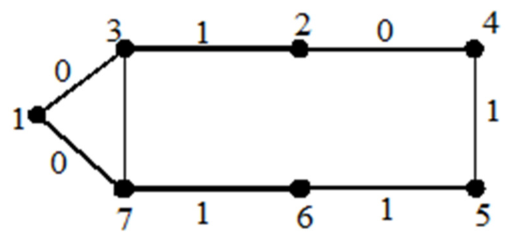

Figure 4

Proposition 2.8

Any Path is a Divided square difference cordial graph.

Proof

Let P_n be the path. Let $v_1, v_2, \ldots, v_n$ be the vertices of the path P_n.

The following tables gives the labels in the vertices of the path graph has Divided square difference cordial graph.

n	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
1	1							
2	1	2						
3	1	3	2					
4	1	3	4	2				
5	1	3	2	4	5			
6	1	3	2	4	5	6		
7	1	3	2	4	5	7	6	
8	1	3	2	4	5	7	8	6

Assume $n > 8$, we can construct the Divided square difference cordial path in the following manner.

Define a map $f: V(P_n) \rightarrow \{1,2,\ldots,n\}$ as follows.

Case 1: $n \equiv 0(mod\ 4)$

We assign the label values in the following way.

For i = 1,5,9,…, $n-3$

$$f(v_i) = i$$
$$f(v_{i+1}) = i + 2$$
$$f(v_{i+2}) = i + 1$$
$$f(v_{i+3}) = i + 3$$

Case 2: $n \equiv 1(mod\ 4)$

We are assigning the label values in the following way.

For i = 1,5,9,…, $n-4$

$$f(v_i) = i$$
$$f(v_{i+1}) = i + 3$$
$$f(v_{i+2}) = i + 1$$
$$f(v_{i+3}) = i + 2$$

And $f(v_n) = n$

Case 3: $n \equiv 2(mod\ 4)$

We are assigning the label values in the following way.

For i = 1,5,9,…, $n-5$

$$f(v_i) = i$$
$$f(v_{i+1}) = i + 2$$
$$f(v_{i+2}) = i + 1$$
$$f(v_{i+3}) = i + 3$$

And $\{f(v_{n-1}) = n - 1\ and\ f(v_n) = n\}$

(Or) $\{f(v_{n-1}) = n\ and\ f(v_n) = n - 1\}$

Case 4: If $n \equiv 3(mod\ 4)$

We are assigning the label values in the following way.

For i = 1,5,9,…, $n-6$

$$f(v_i) = i$$
$$f(v_{i+1}) = i + 2$$
$$f(v_{i+2}) = i + 1$$
$$f(v_{i+3}) = i + 3$$

And $\{f(v_{n-2}) = n - 2\ , f(v_{n-1}) = n\ , f(v_n) = n - 1\}$

(Or) $\{f(v_{n-2}) = n,, f(v_{n-1}) = n - 2\ , f(v_n) = n - 1\ \}$

Hence f is a Divided square difference cordial labeling.

Proposition 2.9

Any Cycle C_n is a Divided square difference cordial graph except $n = 6, 6 + d, 6 + 2d, \ldots$ when $d = 4$.

Proof

Let G be the graph C_n. First we construct the path P_n by Proposition 2.8, and then join the first and last vertices by an edge, we form the cycle C_n, we see that $|e_f(0) - e_f(1)| \leq 1$. Hence C_n is a Divided square difference cordial graph.

Proposition 2.10

The Star graph $K_{1,n}$ is a Divided square difference cordial graph.

Proof

Let v be the central vertex and $v_1, v_2, \dots, v_n$ be the end vertices of the star $K_{1,n}$. Now assign the label 1 to the vertex v and the remaining label to the vertices $v_1, v_2, \dots, v_n$. Then we see that

$$e_f(0) - e_f(1) = \begin{cases} 0, & \text{if } n \text{ is even} \\ 1, & \text{if } n \text{ is odd} \end{cases}$$

Thus $|e_f(0) - e_f(1)| \leq 1$. Hence $K_{1,n}$ is a Divided square difference cordial graph.

Proposition 2.11

The complete bipartite graph $K_{2,n}$ is a Divided square difference cordial graph.

Proof

Let $V = V_1 \cup V_2$ be the bipartition of $K_{2,n}$ such that $V_1 = \{x_1, x_2\}$ and $V_2 = \{v_1, v_2, \dots, v_n\}$.

Now assign the label $1 \leq i \leq n$, to the vertices $v_1, v_2, \dots, v_n$ and $f(x_1) = n + 1$, $f(x_2) = n + 2$. Then it follows that $e_f(0) = e_f(1) =$ n and hence $K_{2,n}$ is a Divided square difference cordial graph.

Example 2.12

Consider $K_{2,8}$.

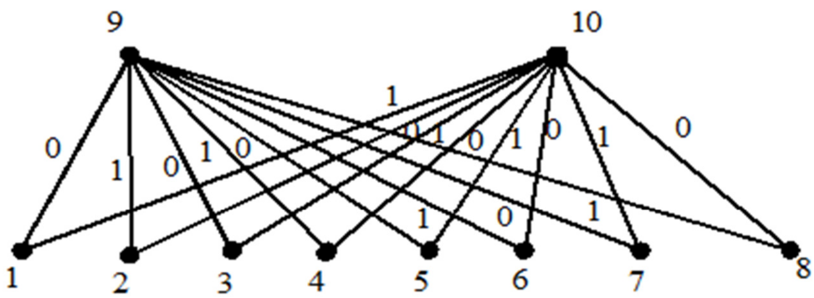

Figure 5

In this graph, $e_f(0) = e_f(1) = 8$.

Proposition 2.13

The graph $P_n + K_1$ is a Divided square difference cordial.

Proof

Let v be the vertices of K_1. Let $v_1, v_2, \dots, v_n$ be the vertices of Divided square difference cordial path P_n. Let $G = P_n + K_1$ be a graph, the vertex set $V(G) = \{v_i, v, 1 \leq i \leq n\}$ and the edge set $E(G) = \{vv_i, v, 1 \le i \le n\}$. Then we assign the label 1 to n in path P_n and $f(v) = n+1$. We construct the Path P_n by Proposition 2.8. Then we can join K_1 to the Path P_n.

Then $|e_f(0) - e_f(1)| \le 1$ and $e_f(1) = e_f(0) + 1$.

Hence $P_n + K_1$ is a Divided square difference cordial graph.

Example 2.14

Consider $P_5 + K_1$

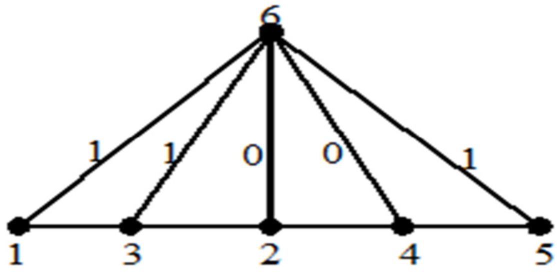

Figure 6

In this graph, $e_f(1) = e_f(0) + 1$.

Proposition 2.15

The double fan $P_n + 2K_1$ is a Divided square difference cordial graph.

Proof

Let y_1, y_2 be the vertices of $2K_1$. Let $v_1, v_2, \dots, v_n$ be the vertices of Divided square difference cordial path P_n. Let $G = P_n + 2K_1$ be a graph, the vertex set $V(G) = \{v_i, y_1, y_2, 1 \le i \le n\}$ and the edge set $E(G) = \{y_1v_i, y_2v_i, for\ all\ i\}$. Now, we assign the label 1 to n in Path P_n and $f(y_1) = n+1$, $f(y_2) = n+2$. We construct the path P_n by Proposition 2.8. Then we join the $2K_1$ to Path P_n. Thus, when n is odd, $e_f(0) = e_f(1)$ and when n is even $e_f(0) = e_f(1) + 1$.

Therefore, $|e_f(0) - e_f(1)| \le 1$.

Hence double fan $P_n + 2K_1$ is a Divided square difference cordial graph.

Example 2.16

Consider $P_3 + 2K_1$

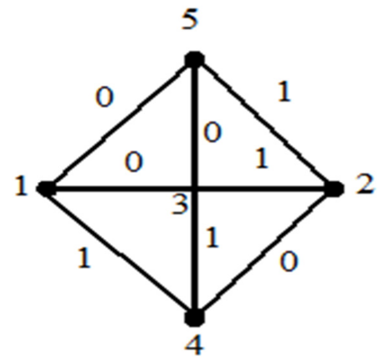

Figure 7

In this graph, $e_f(0) = e_f(1)$.

Consider $P_4 + 2K_1$

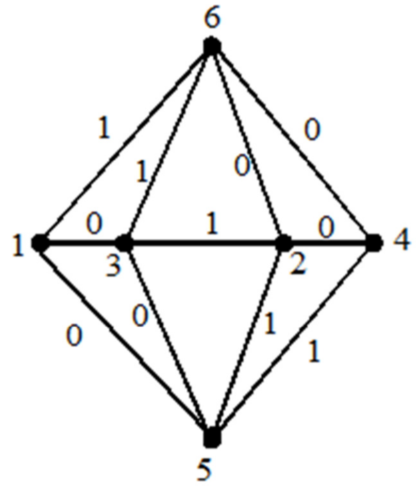

Figure 8

In this graph, $e_f(0) = e_f(1) + 1$.

3. CONCLUSION

In this paper, the concepts of Divided square difference cordial labeling behavior of path, cycle, $K_{1,n}, K_{2,n}, P_n + K_1$ and $P_n + 2K_1$ were discussed.

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