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ENERGY OF EVEN SUM CORDIAL GRAPH

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ABSTRACT

In this paper, we present the concepts of energy and bounds of even sum cordial graph. In this connection, we discussed the concept of Laplacian energy of even sum cordial graph. Several properties of energy of even sum cordial graph are discussed.

Key words: Energy of Even sum cordial graph; Laplacian Energy of Even sum cordial graph; Energy bounds.

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1. INTRODUCTION

By a simple graph for definitions we refer to Harary [6]. The origin of graph labeling can be attributed to Rosa[7]. In [2,3], Cahit introduced the concept of cordial labeling of graph. Energy and Laplacian energy of the graph G was first introduced by Gutman [5, 6] and various bounds are obtained. In this paper, we introduce the concepts of energy and bounds of even sum cordial graph. In this connection, we discussed the concept of Laplacian energy of even sum cordial graph. Several properties of energy of even sum cordial graph are discussed.

2. PRELIMINARIES

Definition 2.1 [4] Even Sum Cordial Graph

Let G= (V, E) be a simple graph and f: V→ {1, 2, 3….|V|} be a bijection. For each edge uv, assign the label 1 if f (u) +f (v) is even and the label 0 otherwise. f is called an even sum cordial labeling if $|e_f(0) - e_f(1)| \leq 1$, where $e_f(1)$ and$e_f(0)$ denote the number of edges labeled with 1 and not labeled with 1 respectively.

Definition 2.2 [1, 5] Energy of a Graph

If G is a graph on n vertices andλ_1,$\lambda_2, \lambda_3, \dots, \lambda_n$ are its Eigen values, then the sum of absolute value of Eigen values of a graph is called energy of a graph G.i.e., E (G) =$\sum_{i=1}^{n}|\lambda_i|$.

Definition 2.3 [9] Hypo-energetic Graph

A graph G with n vertices, whose energy is less than n [i.e., E (G) < n,] is said to be hypo-energetic graph. Otherwise it is called non hypo energetic graph.

Definition 2.4 [4]

- Any path is an even sum cordial graph.
- Any cycle $C_n$is an even sum cordial graph except n = 6, 6 + d, 6 +2d,…

Definition 2.5 [5] Laplacian Energy

The Laplacian matrix of an (m,n) graph G is defined as L(G)=$\Delta(G) - A(G)$, where A is the adjacency matrix and Δ is the diagonal matrix whose diagonal elements are the vertex degree. The Eigen values of Laplacian matrix are$\mu_1, \mu_2, \mu_3, \dots, \mu_n$.

3. ENERGY OF EVEN SUM CORDIAL GRAPH

Definition 3.1

Here we analysis simple graph as it were. Let G = (V, E) be an Even sum cordial graph with n vertices and m edges. We define a matrix of even sum cordial graph G by

$$A = [a_{ij}] = \begin{cases} -1, & \text{if } v_i \text{ and } v_j \text{ are non adjacent} \\ 0, & \text{if } f(v_i) + f(v_j) \text{ is odd or } v_{ii} \\ 1, & \text{if } f(v_i) + f(v_j) \text{ is even} \end{cases}$$

The characteristic polynomial of the labeled matrix of G is defined by

$$\begin{aligned} \emptyset(A(G), \lambda) &= \det[\lambda I - A(G)] \\ &= C_0\lambda^n + C_1\lambda^{n-1} + C_2\lambda^{n-2} + \cdots + C_n \end{aligned}$$

Where I is the unit matrix of order n .The roots of the matrix $A(G)$areλ_1,$\lambda_2, \lambda_3, \dots, \lambda_n$. The roots of the characteristic polynomial$\emptyset(A(G), \lambda)$ are called Eigen values of G.

Throughout this paper, the graph G be an Even sum cordial graph with n vertices and m edges.

Definition 3.2 Energy of even Sum Cordial Graph

The sum of absolute value of Eigen values of even sum cordial graph is called energy of even sum cordial graph G. ie., E(G) =$\sum_{i=1}^{n}|\lambda_i|$,where n denotes the number of vertices of G.

Proposition 3.1

The energy of a graph E(G) = 2 $\sum_{i=1}^{k} \lambda_i$, where k denotes the number of positive Eigen values of G.

Proposition 3.2

Let $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \cdots \geq \lambda_n$ be the Eigen values of G, then $(G) \leq mn$.

Proof:

We know that, each $|\lambda_i| \leq m$, then we adding all absolute values of the Eigen values of G.

$i.e., |\lambda_1| + |\lambda_2| + |\lambda_3| + \cdots + |\lambda_n| \leq m + m + m + \cdots m \ (ntimes)$

$\Rightarrow \sum_{i=1}^{n} |\lambda_i| \leq mn. \Rightarrow E(G) \leq mn$. (1)

Proposition 3.3

If 2m$\geq n$ then inequality $E(G) \leq \frac{2m}{n} + \sqrt{(n-1)(2mn - (\frac{2m}{n})^2)}$holds.

Proof:

Suppose that $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \cdots \geq \lambda_n$ are the Eigen values of even sum cordial graph G. Let,$g(t) = t + \sqrt{(n-1)(2mn - t^2)}$ is a non-increasing function in the interval

$\sqrt{m} \leq t \leq \sqrt{mn}$.Since$\lambda_1$ is the highest Eigen value of the even sum cordial graph G,$\lambda_1 \geq \frac{2m}{n}$

Then each $|\lambda_i| \leq m$ and $E(G) \leq mn$ from (1), we get $\sum_{i=1}^{n} {\lambda_i}^2 \leq mn \leq 2mn$.

Consider, $\sum_{i=1}^{n} {\lambda_i}^2 \leq 2mn$ (2)

From (2), we get $\sum_{i=2}^{n} {\lambda_i}^2 \leq 2mn - {\lambda_1}^2$

$(n-1) \sum_{i=2}^{n} {\lambda_i}^2 \leq (n-1)(2mn - {\lambda_1}^2)$ (3)

By Cauchy-Schwarz inequality, we take this $|\lambda_2|, |\lambda_3|, |\lambda_4|, \ldots, |\lambda_n|$ and 1, 1, 1, 1… (n-1) terms,

We get $(|\lambda_2|.1 + |\lambda_3|.1 + |\lambda_4|.1 + \cdots + |\lambda_n|.1)^2 \leq ({\lambda_2}^2 + {\lambda_3}^2 + {\lambda_4}^2 + \cdots + {\lambda_n}^2)(n-1)$

$(\mathrm{E(G)} - |\lambda_1|)^2 \leq ({\lambda_2}^2 + {\lambda_3}^2 + {\lambda_4}^2 + \cdots + {\lambda_n}^2)(n-1)$.

From (3), we get

$$(\mathrm{E(G)} - |\lambda_1|)^2 \leq ({\lambda_2}^2 + {\lambda_3}^2 + {\lambda_4}^2 + \cdots + {\lambda_n}^2)(n-1) \leq (n-1)(2mn - {\lambda_1}^2)$$

Implies, $(\mathrm{E(G)} - |\lambda_1|)^2 \leq (n-1)(2mn - {\lambda_1}^2)$

Taking square root on both sides, we get $\mathrm{E(G)} - |\lambda_1| \leq \sqrt{(n-1)(2mn - {\lambda_1}^2)}$

$\mathrm{E(G)} \leq \lambda_1 + \sqrt{(n-1)(2mn - {\lambda_1}^2)}$. If $\lambda_1 \geq \frac{2m}{n}$ then $g(\lambda_1) \leq g(\frac{2m}{n})$ is a non-increasing function. Therefore $E(G) \leq \frac{2m}{n} + \sqrt{(n-1)(2mn - (\frac{2m}{n})^2)}$ holds.

Proposition 3.4

If E(G)is the energy of graph G, then $E(G) \leq \frac{n^2(\sqrt{n}+1)}{2}$.

Proof:
Suppose that $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \cdots \geq \lambda_n$ are the Eigen values of even sum cordial graph G. By using the concept of calculus, in Proposition 3.3. We find the R.H.S of the inequality is maximized when$m = \frac{n^2(\sqrt{n}+n)}{4}$. When we substituting the value of m in Proposition **3.3**, we get

$$E(G) \leq \frac{n^2(\sqrt{n}+1)}{2}.$$

Proposition 3.5
If A is a non singular matrix of G then G is a non hypo-energetic graph.

Proof:
We know that the relation between the arithmetic mean and geometric mean.

i.e, A.M $\geq$ G.M. Let A be the matrix of G from Definition **3.1** and$\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_n$ are the Eigen values of G. We take this $|\lambda_1|, |\lambda_2|, |\lambda_3|, |\lambda_4|, \ldots, |\lambda_n|$ terms,

$$\frac{(|\lambda_1| + |\lambda_2| + |\lambda_3| + \cdots + |\lambda_n|)}{n} \geq (|\lambda_1|.|\lambda_2|.|\lambda_3| \ldots |\lambda_n|)^{\frac{1}{n}} = (|\det A(G)|)^{\frac{1}{n}}$$

Since$|\det A(G)| \neq 0$ and $|\det A(G)| \geq 1$ this implies $\frac{(|\lambda_1|+|\lambda_2|+|\lambda_3|+\cdots+|\lambda_n|)}{n} \geq 1$

$$(|\lambda_1| + |\lambda_2| + |\lambda_3| + \cdots + |\lambda_n|) \geq n \Rightarrow E(G) \geq n$$

$\Rightarrow$ G is a non hypo-energetic graph.

Proposition 3.6
Suppose$\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_n$ are the Eigen values of G and m_{ij}denotes elements of the upper or lower triangular matrix of G then $\left[\sum_{i=1}^{n} {\lambda_i}^2\right] = 2\,(\sum_{1 \leq i < j \leq n} {m_{ij}}^2)$.

4. BOUNDS FOR THE ENERGY OF EVEN SUM CORDIAL GRAPH

Proposition 4.1
Let $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \cdots \geq \lambda_n$ be the Eigen values of G. Then$\sqrt{3mn} \leq E(G) \leq mn.$

Proposition 4.2
Suppose $\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_n$ are the Eigen values of G and d (v_i) denotes the degree of each vertex v_i. Then

- E(G) $\leq 2\sum_{i=1}^{n} d(v_i)$
- E(G) $\leq 2m + 2(M + \sqrt{M})$, where M is the maximum degree of the path or cycle even sum cordial graph.

Proposition 4.3
Let G be a graph and d (v_i) denotes the degree of each vertex v_i. If $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \cdots \geq \lambda_n$ are the Eigen values of G then$\lambda_1 \geq \sqrt{\frac{1}{n}\sum_{i=1}^{n}(d(v_i))^2}$. This inequality holds if the graph G is either path or cycle.

Proposition 4.4

Let G be a graph and d (v_i) denotes the degree of each vertex v_i,then

$E(G) \leq \sqrt{\frac{1}{n}\sum_{i=1}^{n}(d(v_i))^2} + \sqrt{(n-1)(2mn - (\frac{\sum_{i=1}^{n}(d(v_i))^2}{n}))}$. This inequality holds if the graph G is either path or cycle.

5. LAPLACIAN ENERGY OF EVEN SUM CORDIAL GRAPH

Definition 5.1

Let G be an Even sum cordial graph with n vertices and m edges and its Laplacian Eigen values are $\mu_1 \geq \mu_2 \geq \mu_3 \geq \cdots \geq \mu_n$ then the Laplacian energy of G is defined by

LE (G) = $\sum_{i=1}^{n}\left|\mu_i - \frac{2m}{n}\right|$and $\gamma_i = \mu_i - \frac{2m}{n}$.

Definition 5.2

Let G be an Even sum cordial graph with n vertices and m edges and its Laplacian Eigen values are $\mu_1 \geq \mu_2 \geq \mu_3 \geq \cdots \geq \mu_n$ then

- $\sum_{i=1}^{n}\mu_i = 2m$
- $\sum_{i=1}^{n}\gamma_i = 0$
- $\sum_{i=1}^{n}{\gamma_i}^2 \geq 2M$, where $M = m + \frac{1}{2}\sum_{i=1}^{n}(d(v_i) - \frac{2m}{n})^2$

Proposition 5.1

Let E(G) and LE(G) be the Energy and Laplacian energy of even sum cordial graph respectively.

Then

- $E(P_n) \leq LE(P_n)$
- $E(C_n) \geq LE(C_n)$ where P_n and C_n are even sum cordial path and cycle graphs.

6. CONCLUSIONS

In this paper, the concepts of energy and bounds of even sum cordial graph have been researched. We have introduced the concept of Laplacian energy of even sum cordial graph. Also some interesting properties of energy of even sum cordial graph are discussed.

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