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COMPOSITE NORM PROPORTIONATE NORMALIZED MINIMUM ERROR ENTROPY ALGORITHM FOR CLUMP SPARSE CHANNEL ESTIMATION

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ABSTRACT

This paper proposes a composite-norm Proportionate Normalized Minimum Error Entropy (CN-PNMEE) algorithm for clump sparse channel estimation. The proposed algorithm imposes a hybrid $l_{2,1}$-norm onto channel coefficients to contemplate the clump sparse feature of the channel. The proposed CN-PNMEE algorithms is developed and studied in detail. Further, the simulations are carried out to prove the efficacy of the proposed algorithm. The exploratory results show that the developed algorithm is superior to existing normalized minimum error entropy (NMEE), Proportionate Normalized Minimum Error Entropy (PNMEE), zero attracting minimum error entropy (ZA-MEE) and residual zero attracting minimum error entropy (RZA-MEE) algorithms for clump sparse channel in the presence of heavy tailed impulsive observation noise.

Key words: Clump sparse channel, composite norm, heavy tailed impulsive noise, minimum error entropy, proportionate adaptive filtering.

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1. INTRODUCTION

Channel estimation is the vital paradigm of adaptive filtering which finds its uses in several applications for instances; echo cancellation, satellite communication, under water acoustic channel estimation, wireless multipath propagation [1-4]. Mean square criterion based normalized least mean square (NLMS) and its variants are most known adaptive algorithms due to ease in developing and simple structures. These conventional MSE based algorithms use second order statistics of error in developing, hence are preferred for Gaussian noise consideration [5].

However, Gaussian noise consideration does not exist in some situations, and then a higher order statistics based adaptive filtering algorithms are considered [6]. Information theoretical learning considers higher order principle in developing algorithms. Some of information theory based criterions are: maximum correntropy, minimum error entropy [6]. Maximum correntropy criterion based adaptive algorithm is optimal for lightly tailed non-Gaussian noise, but Minimum error entropy is well known for heavy tailed impulsive noise [7]. The basic idea of MEE is to extract from data as much information as possible about the unknown systems by minimizing the entropy of error between unknown system output and estimated output [8]. This improves the estimation performance of the system. Further, normalized minimum error entropy (NMEE) algorithm was developed to improve the stability of algorithm due to extent of input signal [9].

In practice, channel are sparse i.e. most of the coefficients are zero or tend to zero and only few have significant contribution. For example, echo channel, satellite channel, HDTV channel are sparse [10]. The estimation behavior of conventional adaptive algorithm can be further improved by taking into account the prior information of sparseness characteristics of the unknown system. The one such algorithm is proportionate LMS (PNLMS) developed by D.L. Duttweiler which improves the estimation behavior by incorporating an individual gain on step size for each coefficient proportional to its magnitude [11]. Larger gains are introduced for dominant coefficients and smaller for negligible coefficients. Therefore, the convergence speed of significant coefficient increases on earlier stage but it is slower down for negligible coefficients at later stage. Moreover, the performance of PNLMS degrades when the system is less sparse. After that, several enhanced PNLMS type algorithms such as μ-law PNLMS (MPNLMS), improved PNLMS (IPNLMS) have been proposed to further improve the performance [12-13]. Based on compressive sensing, several sparsity awared LMS algorithms such as zero-attracting LMS (ZA-LMS), residual zero attracting LMS (RZA-LMS) have been developed for sparse system identification [14]. Under the same concept of sparse penalty, zero attracting MEE, RZA MEE algorithms are invented for sparse channel estimation problems [15]. These algorithms work well for common sparse system into which the dominant coefficients are wide spread along the channel not into clumps. But these algorithms do not perform well for clump sparse system.

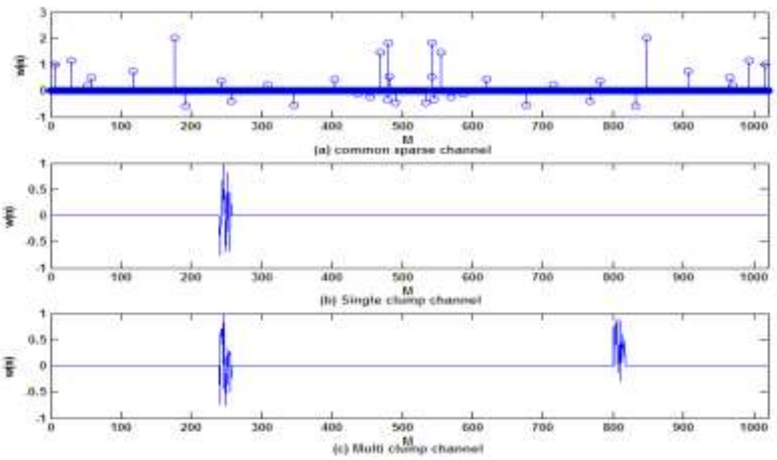

Figure 1 Different types of sparse channel

Based on the arrangement of significant coefficients, the sparse channel can be divided into common sparse channel, single clump sparse channel and multi clump sparse channel.

However, the network echo path comes under the category of one clump sparse channel owing to distribution of significant coefficients into a chunk and the echo path of satellite communication falls under multiple clump sparse channel due to huge amount of packet delay variation, and loading and encoding in networking [16]. To consider the multi clump sparse channel, block sparse LMS algorithm was invented by adding hybrid $l_{2,0}$ in to cost function of conventional LMS algorithm [17]. Further, several group sparse adaptive algorithms have been developed by incorporating mixed-norm constraint on to channel coefficients to improve the performance in multi clump sparse channel [18-20].

In this work, a composite norm proportionate normalized MEE based adaptive algorithm is developed for clump sparse channel. The composite norm considers the prior information about the arrangement of dominant coefficients into single clump or multi clump. The proposed CN-PNMEE is developed by incorporating composite $l_{2,1}$ into cost function of PNMEE algorithm. Thereafter, simulations are performed to prove the credibleness of proposed algorithm for clump sparse channel in the presence of impulsive observation noise. The rest of the paper is organized as follows. In Section 2, reviews the Renyi's Entropy and section 3 derives PNMEE algorithm. In Section 4, we derive the proposed CN- PNMEE algorithm. In Section 5, simulation results are carried out to excel the performance of the proposed algorithm. Finally, In Section 6, conclusion is drawn.

2. RENYI'S ENTROPY

Entropy measures the amount of information from available data samples of random variable. Consider X as random variable and f(x) as its probability density function (pdf), the quadratic entropy (Renyi's entropy) of random variable X can be written as [8]:

$$\mathrm{H_X} = -\,log\ \int f^2(X)\,\mathrm{d}X \tag{1}$$

Due the lack of available data samples of random variable X in practice, an estimation of f(X) from available samples is considered using Parzen window [9]. Hence, the estimation of pdf can be written as:

$$\hat{f}(\mathrm{X}) = \frac{1}{N}\sum_{j=1}^{N} g_\sigma\,(\mathrm{X}-\mathrm{X(j)}) \tag{2}$$

Where, N is number of available samples and g_σ is kernel function having bandwidth σ. The most popular kernel used is Gaussian kernel defined as:

$$g_\sigma\,(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{\frac{-x^2}{2\sigma^2}} \tag{3}$$

So we can write estimated Renyi's entropy $\widehat{\mathrm{H}}_{\mathrm{X}}$ as :

$$\widehat{\mathrm{H}}_X = -\,log\ \int \hat{f}^2(X)\,\mathrm{d}X = -\,log\,\frac{1}{N^2}\ \int\Big[\sum_{j=1}^{N} g_\sigma\ (\mathrm{X}-\mathrm{X(j)})\,\Big]^2\,\mathrm{dX}$$

$$= -\,log\,\frac{1}{N^2}\ \sum_{j=1}^{N}\sum_{k=1}^{N}\int g_\sigma\,(X-X(j)\,g_\sigma\,(X-X(k)\,dX$$

$$= -\,log\,\frac{1}{N^2}\ \sum_{j=1}^{N}\sum_{k=1}^{N} g_{\sigma\sqrt{2}}(X(i)-X(j)\,) = -\,log\ \ S(\boldsymbol{X}) \tag{4}$$

where $\mathrm{S}(\mathbf{X}) = \frac{1}{N^2}\ \sum_{j=1}^{N}\sum_{k=1}^{N} g_{\sigma\sqrt{2}}(X(i)-X(j)\,)$, is defined as information potential.

3. PROPORTIONATE NORMALIZED MINIMUM ERROR ENTROPY (PNMEE)

Consider $\mathbf{x}(n)= [x(n)\ x(n-1)\ x(n-2) \ldots\ldots\ldots\ldots x(n-M+1)]^T$ as input to both unknown channel and adaptive filter. Considering $\mathbf{h}(n) = [h_0(n), h_1(n), h_2(n), \ldots\ h_{M-1}(n)]^T$ as unknown channel impulse response, we can write the reference output of unidentified channel as:

$$d(n) = \mathbf{x}^{\boldsymbol{T}}(n)\boldsymbol{h}(n) + v(n) \tag{5}$$

where $v(n)$ is measurement noise with zero mean and variance σ_v^2 which is independent of input $\mathbf{x}(n)$. Now the output of adaptive filter with $\hat{\boldsymbol{h}}(n)$ as estimated impulse response can be written as:

$$y(n) = \mathbf{x}^{\boldsymbol{T}}(n)\, \hat{\boldsymbol{h}}(n-1) \tag{6}$$

The instantaneous estimation error $e(n)$ is represented as:

$$e(n) = d(n) - y(n) \tag{7}$$

The PNMEE algorithm just like PNLMS uses a posteriori error based cost function. The aim of PNMEE is to minimize the entropy of a posteriori error $e_p(n) = d(n) - \mathbf{x}^{\boldsymbol{T}}(n)\hat{\boldsymbol{h}}(n)$.

The estimated Renyi's entropy $\widehat{\mathrm{H}}_{\mathrm{R}}$ can be represented as [8] :

$$\widehat{\mathrm{H}}_{\mathrm{R}} = -\log \frac{1}{N^2} \sum_{i=1}^{N}\sum_{j=1}^{N} g_{\sigma\sqrt{2}}(e_p(i) - e_p(j)) = -\log\ S(\boldsymbol{e}_p) \tag{8}$$

Where $S(\boldsymbol{e_p}) = \frac{1}{N^2}\sum_{j=1}^{N}\sum_{k=1}^{N} g_{\sigma\sqrt{2}}(e_p(i) - e_p(j))$, is defined as information potential .Thus, it is clear that the objective of PNMEE principle is to minimize error entropy which is equivalent to maximizing information potential and $\boldsymbol{e}_p = [e_p(1), e_p(2), \ldots, e_p(N)]$

As it is clear that $S(\mathbf{0}) \geq S(\boldsymbol{e}_p(n))$, hence the cost function of PNMEE can be written as [9]:

Therefore, the cost function of PNMEE becomes:

$$J_{PNMEE}(\hat{\boldsymbol{h}}) = \left\|\hat{\boldsymbol{h}}(n) - \hat{\boldsymbol{h}}(n-1)\right\|^2_{\mathbf{G}^{-1}(n-1)} + \lambda\,(S(\boldsymbol{e}_p(n) - S(\mathbf{0})) \tag{9}$$

We can write as:

$$J_{PNMEE}(\hat{\boldsymbol{h}}) = \left(|\hat{\boldsymbol{h}}(n) - \hat{\boldsymbol{h}}(n-1)|\right)^T\ \mathbf{G}^{-1}(n-1)\left(|\hat{\boldsymbol{h}}(n) - \hat{\boldsymbol{h}}(n-1)|\right) + \lambda(S(\boldsymbol{e}_p(n)) - S(\mathbf{0})) \tag{10}$$

where $\mathbf{G}(n)$ is a gain control matrix that manages the step size for each coefficient.

$$\frac{\partial J_{PNMEE}(\hat{\boldsymbol{h}})}{\partial \hat{\boldsymbol{h}}(n)} = 2\ \mathbf{G}^{-1}(n-1)\left(\hat{\boldsymbol{h}}(n) - \hat{\boldsymbol{h}}(n-1)\right) + \lambda\nabla S(\boldsymbol{e}_p(n)) \tag{11}$$

$$\frac{\partial J_{PNMEE}(\hat{\boldsymbol{h}})}{\partial \lambda} = S(\boldsymbol{e}_p(n)) - \mathrm{S}(\boldsymbol{0}) \quad (12)$$

By putting $\frac{\partial J_{PNMEE}(\hat{\boldsymbol{h}})}{\partial \hat{\boldsymbol{h}}(n)} = 0 \; and \; \frac{\partial J_{PNMEE}(\hat{\boldsymbol{h}})}{\partial \lambda} = 0$

$$\lambda = \frac{2e(n)\mathbf{G}(\mathrm{n-1}\nabla S(\boldsymbol{e}_p(n))}{[\nabla S(\boldsymbol{e}_p(n))]^T \mathbf{x}(\mathrm{n})} \quad (13)$$

where $\nabla S(\boldsymbol{e}_p(n)) = \frac{\partial S(\boldsymbol{e}_p(n))}{\partial \hat{\boldsymbol{h}}(n)}$ (14)

$$\nabla S(\boldsymbol{e}_p(n)) = \frac{\partial}{\partial \hat{h}(n)} \left[\frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} g_{\sigma\sqrt{2}}(e_p(i) - e_p(j))\right]$$

$$= \frac{1}{2N^2} \left[\sum_{i=1}^{N} \sum_{j=1}^{N} g_{\sigma\sqrt{2}}(e_p(i) - e_p(j)) \left(e_p(i) - e_p(j)\right) (x(i) - x(j))\right] \quad (15)$$

Hence the weight update equation of PNMEE becomes:

$$\hat{\boldsymbol{h}}(n) = \hat{\boldsymbol{h}}(n-1) + \mu \frac{e(n)\mathbf{G}(\mathrm{n-1})\nabla S(\boldsymbol{e}_p(n))}{[\nabla S(\boldsymbol{e}_p(n))]^T \mathbf{G}(\mathrm{n-1})\mathbf{x}(\mathrm{n}) + \xi_{PNMEE}} \quad (16)$$

where μ is the step size to control change in each iteration and ξ_{PNMEE} is regularizer to prevent the overflow of division by zero; and **G**(n-1) is the diagonal matrix of dimension MXM ,which controls the step size can be written as:

$$\mathbf{G}(\mathrm{n-1}) = \mathrm{diag}\{g_0(n-1), g_1(n-1), \ldots, g_{M-1}(n-1)\} \quad (17)$$

The elements of control matrix **G**(n) in this algorithm can be written as:

$$g_\ell(n) = \frac{\gamma_\ell(n)}{\sum_{i=0}^{M-1} \gamma_i(n)} \quad (18)$$

$$\gamma_\ell(k) = \max\,[\rho \times \max\,(q, |\hat{h}_{\mathbf{0}}|, |\hat{h}_1|, \ldots, |\hat{h}_{M-1}|), |\hat{h}_\ell|] \quad (19)$$

With ℓ=0,1,…, M-1

The parameter ρ helps the coefficients to be in consideration even when their value is much smaller than the highest coefficient and parameter q helps the coefficients to be in consideration even when $\hat{h}_\ell$ is at initial stage i.e. $\hat{h}_\ell(0)=0$.

4. PROPOSED COMPOSITE NORM PNMEE (CN-PNMEE) ALGORITHM

To consider the clump sparseness characteristics of channel, CN-PNMEE algorithm is proposed. The proposed algorithm is developed by adding composite $l_{2,1}$ norm to cost function of PNMEE algorithm.

The cost function of CN-PNMEE algorithm can be written as:

$$J_{CN-PNMEE}(\hat{\boldsymbol{h}}) = \left\|\hat{\boldsymbol{h}}(n) - \hat{\boldsymbol{h}}(n-1)\right\|^2_{2,1\; G^{-1}(n-1)} + \lambda\,(S(\boldsymbol{e}_p(n)) - \mathrm{S}(\boldsymbol{0})) \quad (20)$$

Under limiting condition of norm, the above cost function becomes:

$$J_{CN-PNMEE}(\hat{\boldsymbol{h}}) = \left(\|\hat{\boldsymbol{h}}(n)\|_{2,1} - \|\hat{\boldsymbol{h}}(n-1)\|_{2,1}\right)^T \; G^{-1}(n-1)\left(\|\hat{\boldsymbol{h}}(n)\|_{2,1} - \|\hat{\boldsymbol{h}}(n) - 1\|_{2,1}\right) + \lambda\,(S(\boldsymbol{e}_p(n)) - \mathrm{S}(\boldsymbol{0})) \quad (21)$$

where

$$\|\hat{\boldsymbol{h}}(n)\|_{2,1} = \left\|\begin{bmatrix}\|\hat{\boldsymbol{h}}_1\| \\ \|\hat{\boldsymbol{h}}_2\| \\ . \\ . \\ . \\ \|\hat{\boldsymbol{h}}_C\|\end{bmatrix}\right\|_1 \quad (22)$$

Where C is number of clumps and procured by dividing total number of coefficients M of unknown system by number of coefficients L present in each clump i.e. C=M/L and $\|\mathbf{x}\|_1$ denotes ℓ_1 –norm .

$$\|\mathrm{x}\|_1 = \textstyle\sum_i |x|_i \quad (23)$$

$$\|\hat{\boldsymbol{h}}(n)\|_{2,1} = \textstyle\sum_{i=1}^{C} \|\hat{\boldsymbol{h}}_i(n)\| \quad (24)$$

$$\frac{\partial\|\hat{\boldsymbol{h}}(n)\|_{2,1}}{\partial\hat{\boldsymbol{h}}(n)} = \left[\frac{\partial\|\hat{\boldsymbol{h}}(n)\|_{2,1}}{\partial\hat{\mathrm{h}}_0}, \frac{\partial\|\hat{\boldsymbol{h}}(n)\|_{2,1}}{\partial\hat{h}_2}, \frac{\partial\|\hat{\boldsymbol{h}}(n)\|_{2,1}}{\partial\hat{h}_3}, \dots, \frac{\partial\|\hat{\boldsymbol{h}}(n)\|_{2,1}}{\partial\hat{h}_{M-1}}\right]^T \quad (25)$$

Considering jth coefficient is allied to rth clump of system response, hence we let:

$$\frac{\partial\|\hat{\boldsymbol{h}}(n)\|_{2,1}}{\partial\hat{h}_j} = \frac{\partial}{\partial\hat{h}_j}(\hat{h}^2_{(r-1)L+1} + \hat{h}^2_{(r-1)L+2} + \hat{h}^2_{(r-1)L+3}, \dots, \hat{h}^2_{rL})^{\frac{1}{2}} \quad (26)$$

$$= \frac{1}{2}(\hat{h}^2_{(r-1)L+1} + \hat{h}^2_{(r-1)L+2} + \hat{h}^2_{(r-1)L+3}, \dots, \hat{h}^2_{rL})^{-\frac{1}{2}}\; 2\hat{h}_j = \frac{\hat{h}_j}{\|\hat{\boldsymbol{h}}_r\|_2} \quad (27)$$

Using Lagrange Multiplier method

$$\frac{\partial J_{CN-PNMEE}(\hat{\boldsymbol{h}})}{\partial\hat{\boldsymbol{h}}(n)} = 0 \quad (28)$$

And $\frac{\partial J_{CN-PNMEE}(\hat{\boldsymbol{h}})}{\partial\lambda} = 0$ (29)

We get

$$\frac{\partial J_{CN-PNMEE}(\hat{\boldsymbol{h}})}{\partial \hat{\boldsymbol{h}}(n)} = \mathbf{G}^{-1}(\mathrm{n}-1)\left[\frac{\hat{\boldsymbol{h}}(n)}{\|\hat{\boldsymbol{h}}_r\|} - \frac{\hat{\boldsymbol{h}}(n-1)}{\|\hat{\boldsymbol{h}}_r\|}\right] + \lambda\nabla\left(S(\boldsymbol{e}_p(n)\right) = 0 \quad (30)$$

Pre-multiplying both sides of (36) by **G**(n-1), we let:

$$\left[\frac{\hat{\boldsymbol{h}}(n)}{\|\hat{\boldsymbol{h}}_r\|} - \frac{\hat{\boldsymbol{h}}(n-1)}{\|\hat{\boldsymbol{h}}_r\|}\right] + \lambda\,\mathbf{G}(\mathrm{n}-1)\nabla S\left(\boldsymbol{e}_p(n)\right) = 0 \quad (31)$$

Pre multiplying (31) both sides by $\mathbf{x}^T(n)$, we have

$$\left[\mathbf{x}^T(n)\ \frac{\hat{\boldsymbol{h}}(n)}{\|\hat{\boldsymbol{h}}_r\|} - \mathbf{x}^T(n)\frac{\hat{\boldsymbol{h}}(n-1)}{\|\hat{\boldsymbol{h}}_r\|}\right] + \lambda\mathbf{x}^T(n)\ \mathbf{G}(\mathrm{n}-1)\nabla S(\boldsymbol{e}_p(n)) = 0 \quad (32)$$

$$\left[\mathbf{x}^T(n)\ \hat{\boldsymbol{h}}(n) - \mathbf{x}^T(n)\hat{\boldsymbol{h}}(n-1)\right] + \lambda\left\|\hat{\boldsymbol{h}}_r\right\|\mathbf{x}^T(n)\ \mathbf{G}(\mathrm{n}-1)\nabla S(\boldsymbol{e}_p(n)) = 0 \quad (33)$$

and from (29), we have:

$$S(\boldsymbol{e}_p(n)) = S(\mathbf{0}) \quad (34)$$

Therefore,

$$e_p(n) = 0$$
$$\mathrm{d(n)} - \mathbf{x}^{\boldsymbol{T}}(\mathrm{n})\hat{\boldsymbol{h}}(\mathrm{n}) = 0$$
$$\mathrm{d(n)} = \mathbf{x}^{\boldsymbol{T}}(\mathrm{n})\hat{\boldsymbol{h}}(\mathrm{n}) \quad (35)$$

Hence, from (33) and (35), we have:

$$\mathrm{e(n)} + \lambda\left\|\hat{\boldsymbol{h}}_r\right\|\mathbf{x}^T(n)\ \mathrm{G(n)}\nabla S(\boldsymbol{e}_p(n)) = 0 \quad (36)$$

$$\lambda = -\frac{\left[\|\hat{\boldsymbol{h}}_r(n)\|\right]^{-1} e(n)}{\mathbf{x}^T(n)\mathbf{G}(\mathrm{n}-1)\nabla S(\boldsymbol{e}_p(n))} \quad (37)$$

Herein, the diagonal matrix **G**(n-1) is described as:

$$\mathbf{G}(\mathrm{n}-1) = \mathrm{diag}\ \{g_1(n-1)\ \mathbf{1}_\mathrm{L},\ g_2(n-1)\mathbf{1}_\mathrm{L},\ \cdots,\ g_C(n-1)\ \mathbf{1}_\mathrm{L}\} \quad (38)$$

$$g_r(n) = \frac{\gamma_\mathrm{r}(n)}{\sum_{i=1}^{C}\gamma_i(n)} \quad (39)$$

$$\gamma_r(n) = \max\ [\rho \times \max\ (\mathrm{q},\ f_1(\hat{h}),\ f_2(\hat{h}),\ \ldots,\ f_C(\hat{h}))\ ,\ f_r(\hat{h})] \quad (40)$$

Where $f_r(\widehat{w}) = \left\|\hat{\boldsymbol{h}}_r(n)\right\|$ (41)

Hence, the new update equation becomes:

$$\hat{\boldsymbol{h}}(n) = \hat{\boldsymbol{h}}(n-1) + \mu \frac{e(n)\mathbf{G}(\mathrm{n}-1)\nabla \mathrm{S}(e_p(n))}{\mathbf{x}^T(n)\mathbf{G}(\mathrm{n}-1)\nabla \mathrm{S}\left(e_p(n)\right)} \quad (42)$$

Further a regularizer, $\xi_{CN-PNMEE}$ is added in the denominator of (42) to prevent the overflow due to division by zero.

$$\hat{\boldsymbol{h}}(n) = \hat{\boldsymbol{h}}(n-1) + \mu \frac{e(n)\mathbf{G}(\mathrm{n}-1)\nabla \mathrm{S}(e_p(n))}{\mathbf{x}^T(n)\mathbf{G}(\mathrm{n}-1)\nabla \mathrm{S}\left(e_p(n)\right)+\xi_{CN-PNMEE}} \quad (43)$$

However, a posteriori error $e_p(n)$ is replaced by a priori error e(n) to reduce the computational complexity.
Therefore, (43) can be written as:

$$\hat{\boldsymbol{h}}(n) = \hat{\boldsymbol{h}}(n-1) + \mu \frac{e(n)\mathbf{G}(\mathrm{n}-1)\nabla \mathrm{S}(e_p(n))}{\mathbf{x}^T(n)\mathbf{G}(\mathrm{n}-1)\nabla S\left(e(n)\right)+\xi_{CN-PNMEE}} \quad (44)$$

5. SIMULATION RESULTS

In the above section, we have derived the CN-PNMEE algorithm that takes into account the clump sparsity form of the channel by using $\ell_{2,1}$ norm on coefficients. Further, the performance of proposed CN-PNMEE algorithm is verified by conducting several experiments and is compared to NMEE, PNMEE, ZA-MEE, RZA-MEE algorithms.

In all experiments, the length of channel is L=1024. The unknown channel length and adaptive filter length are considered to be same. The simulations are carried out considering three types of input signal.

- Gaussian Input
- Colored Input
- Speech signal

The colored input is generated by passing Gaussian input through AR process whose transfer function, $H(z) = \frac{1}{1-0.9z^{-1}}$. The sampling frequency of speech signal is taken 8 Khz.

The measurement noise is considered as impulsive. It is taken as alpha stable distribution $\mathrm{V}(\alpha, \beta, \gamma, \delta)$ i.e. [20]. We have taken measurement noise as: V(1.3,0,0.2,0).

The regularizer constants $\xi_{NMEE} = 0.01$ and $\xi_{PNMEE} = \xi_{CN-PNMEE} = \frac{1}{M}\xi_{NMEE}$ are taken in all simulations. The kernel width $\sigma = 0.7$ is adopted here.

To consider the clump sparsity, first we have consider one clump channel and then two clump channel is taken for simulation purpose. In figure 1, in one clump channel, the dominant coefficients are in dispersed in (241, 256) and in second type of channel, the active coefficients are in (241, 256) and (801, 816).

In the first experiment, we demonstrate the consequence of clump size L=[4, 8,16,32,64,128] on the tracking behavior of CN-PNMEE algorithm. The channel is considered single clump and the input noise is taken as Gaussian. As the clump size M increases from 8 to 16, the estimation performance improves after that it starts deteriorating. The regularizer constants $\xi_{NMEE} = 0.01$ and $\xi_{PNMEE} = \xi_{CN-PNMEE} = \frac{1}{M}\xi_{NMEE}$ are taken in all simulations. The kernel width $\sigma = 0.7$ is adopted here. The step sizes considered here are:$\mu_{ZA-MEE} = \mu_{ZA-MEE} = 0.001$, $\mu_{NMEE} = 0.6$, $\mu_{CN-PNMEE} = \mu_{PNMEE} = 0.2$. Fig.1 shows the consequence of clump size M on the performance of CN-PNMEE algorithm for colored input and two clumps sparse channel. The convergence speed and NMSD for L=16 is superior to other values of M.

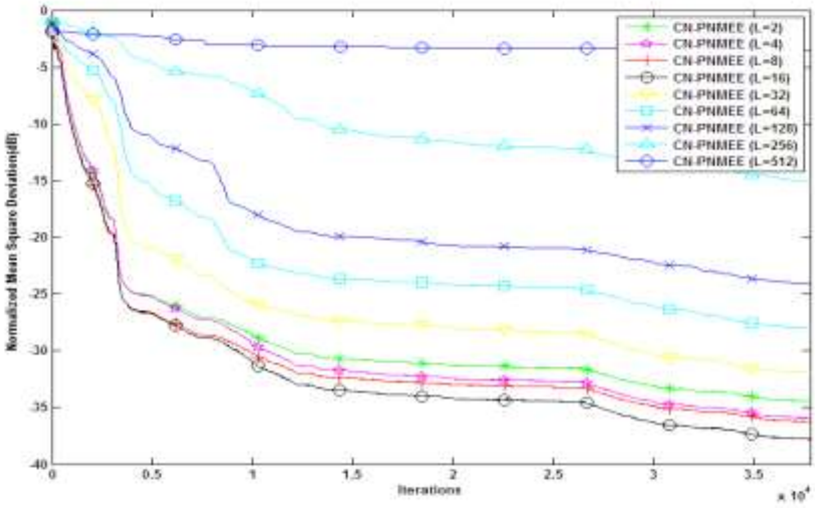

Figure 2 Consequence of clump size L on the tracking behavior of CN-PNMEE

In the next experiment, we compare the estimation performance with NMEE, PNMEE, ZA-MEE, RZA-MEE algorithms for different types of input.

For each type of input, we have considered both single clump channel and two clumps channel. The other parameters are adopted same as in figure 2.

Figs. 3(a), 3(b) and 3(c) show the estimation performance for Gaussian, colored and Speech input signals. It is obvious form fig 3 that the proposed algorithm is robust against any type of input and excels for both single clump and double clump sparse channel estimation.

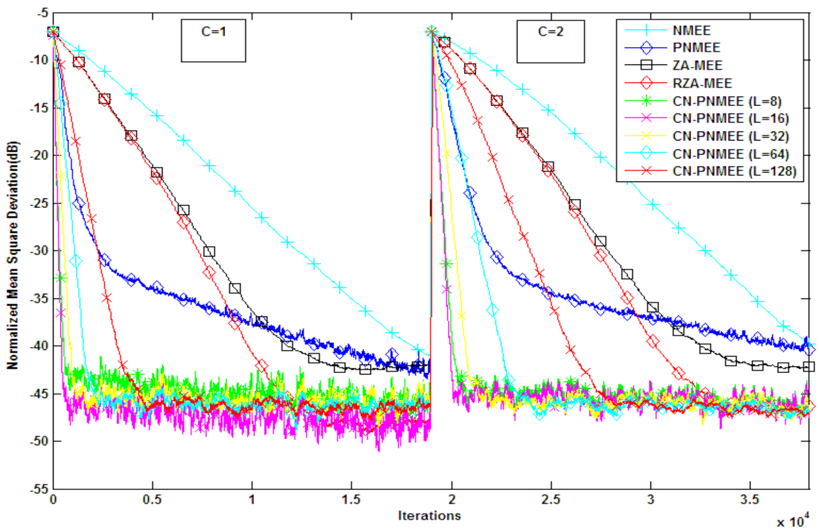

Figure 3(a) Comparison of tracking behavior of CN-PNMEE algorithm with cited algorithms for Gaussian input

From figs. 3(a), 3(b) and 3(c), it is clear that the proposed CN-PNMEE algorithm perform better than other mentioned algorithms for every input in case of both single and double clump channels. However, the performance is superior for L=16 and worst for L=128.

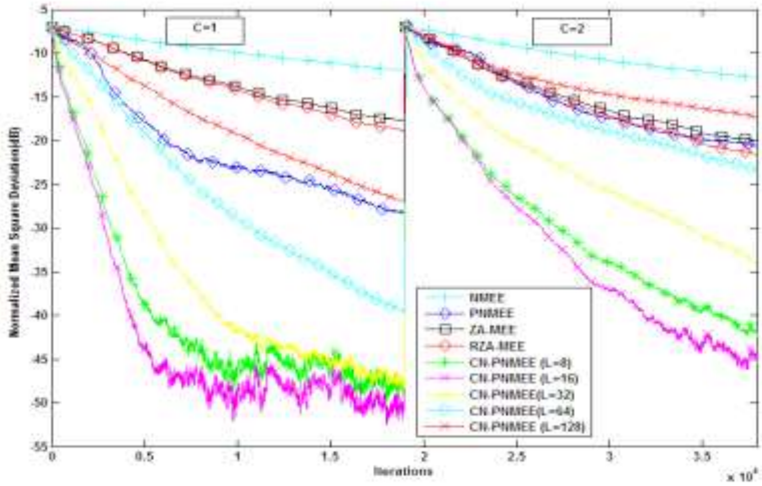

Figure 3(b) Comparison of tracking behavior of CN-PNMEE algorithm with cited algorithms for Colored input

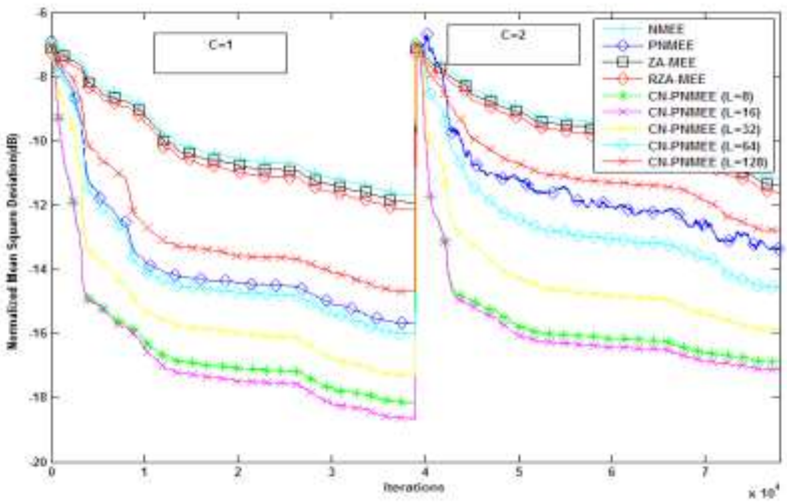

Figure 3(c) Comparison of tracking behavior of CN-PNMEE algorithm with cited algorithms for Speech input signal

In the next experiment, we compare the performance of proposed CN-PNMEE with $l_{2,0}$-PNLMS and mixed norm constrained proportionate normalized least mean fourth (MNC-PNLMF) algorithms for L=16, C=2 and colored input. First we have taken Gaussian observation noise and then the simulations are performed for alpha stable noise. For Gaussian observation noise, the SNR=20 dB is considered. The step size μ=0.01, μ=0.2 and μ=0.3 are taken for MNC-PNLMF, $l_{2,0}$-PNLMS, CN-PNMEE algorithms respectively for Gaussian observation noise. The other parameters are same as in above experiments.

From figure 4(a), the performance of the proposed CN-PNMEE algorithm is better than other cited algorithms for Gaussian observation noise.

However, MNC-PNLMF $l_{2,0}$-PNLMS algorithms do not converge for alpha stable observation noise. From figure 4 (b), it is clear the proposed CN-PNMEE algorithm perform better than other mentioned algorithms for heavy tail impulsive noise.

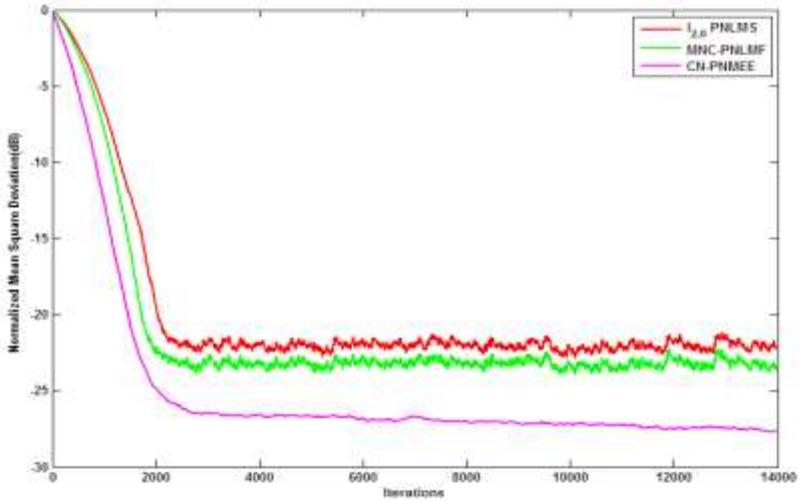

Figure 4(a) Comparison of the performance of proposed CN-PNMEE with $l_{2,0}$-PNLMS and mixed norm constrained proportionate normalized least mean fourth (MNC-PNLMF) algorithms in the presence of Gaussian observation noise

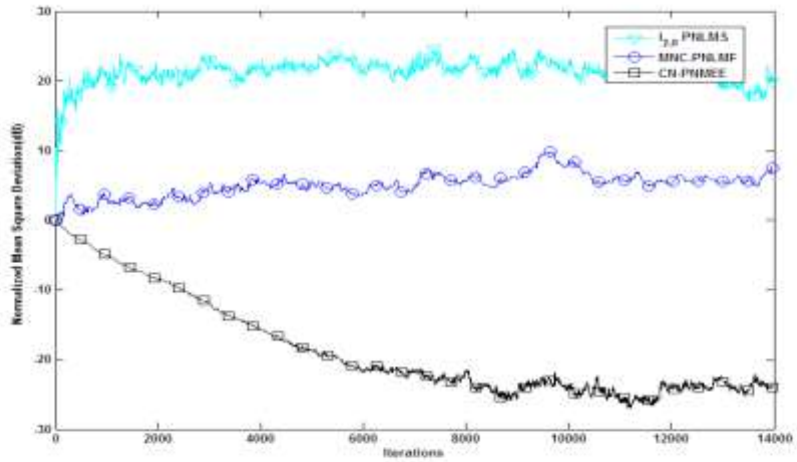

Figure 4(b) Comparison of the performance of proposed CN-PNMEE with $l_{2,0}$-PNLMS and mixed norm constrained proportionate normalized least mean fourth (MNC-PNLMF) algorithms in the presence of alpha stable observation noise

6. CONCLUSION

A new composite norm proportionate normalized minimum error entropy (CN-PNMEE) algorithm has been developed and investigated for clump sparse channel estimation paradigm. The proposed algorithm is established by incorporating $l_{2,1}$ norm into cost function of PNMEE algorithm to positively exploit the clump sparse information of channel which is available in advance. The performance of the proposed algorithm is tested for different types of input signal. However, the proposed algorithm excels in clump sparse channel in the presence of heavy tailed impulsive noise.

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