MANIPULABILITY INDEX OF A PARALLEL ROBOT MANIPULATOR

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ABSTRACT

The manipulability index is used to quantify manipulators velocity transmission capabilities or the dexterity of robot. The index is a measure of manipulating ability of robotic mechanisms in positioning and orienting the end effectors. The index allows to quantify the proximity of the robot to a singularity, which is ideal for identifying paths of welding with continuous and soft movements at the robot joints. A university of Maryland manipulator is considered in the present paper. Manipulability index values are determined for the selected structures. A MATLAB code is developed and used for the analysis. The results are graphically presented.

Keywords: Manipulability index, Parallel Robotic Manipulator, Condition Number, Force Isotropy, Performance Measures

I. INTRODUCTION

Karl Gotlih, Denis Kovac, Tomaz Vuherer and Simon Brezovnik (1) presented the planning process of robotized production cells, a useful tool for correct decision where to put the robot in the cell with respect to technologically conditioned tasks. The developed tool enables 3D representation of the velocity anisotropy of the industrial robot in the workspace, especially in welding robots. The velocity anisotropy is defined as the normalized length of the shortest velocity ellipsoid axes, which can be constructed for any position of robot in its TCP. Zacharias F, Borst C and Hirzinger G (2) has developed the representation called the capability map to capture the manipulator capabilities in its workspace. It is especially useful for two-armed humanoid robots. Using the capability map to solve the problem of finding an approach and grasp direction that facilitates the work of path planners is possible. Furthermore, the maps could perhaps be used to develop Cartesian space path planners or evaluate and compare robot designs. Ottaviano E, Cecarelli M, and Husty M. (3) presented a classification of industrial 3R manipulators based on kinematic properties of the workspace. A level-set reconstruction was used to analyse workspace topology by using algebraic expressions, which helped to classify industrial 3R manipulators and to identify design conditions for avoiding singularities in the workspace.
Ceccarelli M. (4) proposed an algebraic formulation for the workspace boundary N-revolute joint open chain manipulators, as a generalization of a procedure first proposed for 3R manipulators and then extended specifically up to 5R manipulators. He refined the basic formulation reported earlier. A recursive process for the generation of the workspace volume as a solid of revolution has been recognized and some results from the common topology of torus, ring and hyper-ring has been deduced. Chiu SL (5) studied the compatibility of the manipulator postures with respect to task requirements. He focused on velocity and force transmission ratios, viewing the manipulator as a mechanical transformer. An index has been proposed by him for measuring the compatibility of manipulator postures with respect to a generalized task description. The index maximization can be utilized for manipulator redundancy as well as non-redundant manipulators.

Yoshikawa T (7) studied the properties of the manipulability as a measure of manipulating ability of robotic arms in positioning and orienting end-effectors. Best postures of various types of manipulators and an articulated robot finger has been obtained by utilization of the manipulability measure. The best postures obtained have some resemblance to those taken by human arms and fingers. Resolved motion means that the motions of the various motors are combined and resolved into separately controllable hand motions along world coordinates. The implication is that several motors must run simultaneously at different and time-varying rates in order to achieve steady hand motion along one world coordinate. The mathematics of multi-degree-of-freedom manipulators and prostheses has analyzed by Whitney DE. (8). A redundant arm can be "programmed" to obey certain useful and relevant constraints during motion, constraints which would be difficult to obey with conventional one-to-one rate control. Ming J. Tsai and Yee H. Chiou (9) studied the manipulability of robot arms relative to the singularity of its Jacobian matrix. Manipulability is used as criterion function to be maximized in the control of kinematically redundant manipulators. The manipulability is optimized during the operation the manipulator. Singularities have been reduced by the use of manipulability in joint rate control algorithm. The joint availability function can detect the joint limits and by optimizing it the robot can change aspect before a joint limit is reached.

Francois J, Chedmail P, Hascoet JY (10) developed the optimal scheduling of the functional points of a trajectory, in order to minimize the trajectory run time of the robot. For example different task points in spot welding operation. This helps in the minimization of total productive time of a robot. The method suits well to the two or three degree-of-freedom (DOF) robots. However, it is very difficult to generalize it to more degrees of freedom because of the increasing computer time cost.

II. PERFORMANCE INDEX (MANIPULABILITY)

The concept of manipulability of a manipulator was introduced by Yoshikawa. The manipulability is defined as the square root of the determinant of the product of the manipulator Jacobian by its transpose, i.e.,

$$w = \sqrt{\det(JJ^T)}$$

The manipulability $w$ is equal to the absolute value of the determinant of the Jacobian in case of a square Jacobian. Using the singular value decomposition the manipulability can be written as follows:

$$w = \pi(\sigma_i)$$

where $\sigma_i$ are the singular values of the Jacobian matrix $J$. 
The manipulability index will have different values for the different used units. Therefore, it is convenient to use the manipulability index \( \hat{\omega} \) using the normalized Jacobian \( \hat{J} \).

\[
\hat{J} = T_v J(q) T_q^{-1}
\]

\[
T_v = \text{diag} \left( \frac{1}{v_1 \text{ max}}, \frac{1}{v_2 \text{ max}}, ..., \frac{1}{v_m \text{ max}} \right)
\]

\[
T_q = \text{diag} \left( \frac{1}{q_1 \text{ max}}, \frac{1}{q_2 \text{ max}}, ..., \frac{1}{q_n \text{ max}} \right)
\]

where \( v_j \text{ max} \) is maximum angular or linear velocity for each element of the end-effector velocity \( v_j \).

\( q_i \text{ max} \) is maximum angular or linear velocity of joint \( i \).

\[
\hat{\omega} = \sqrt{\text{det} (\hat{J}(q)\hat{J}^T(q))}
\]

For a square Jacobian matrix,

\[
\hat{\omega} = \text{det} T_v \text{det} J(q) \text{det} T_q^{-1}
\]

\[
= [\prod_{i=1}^{m}(q_{i \text{ max}}/v_{i \text{ max}})]\omega
\]

When a robot is in a singular position, the manipulability at the robot is zero. Each robot posture may have a different value of manipulability. The manipulability is at the maximum around the middle of the workspace. The manipulability is larger when the angles of revolute joints are near 90° and prismatic joints are at their zero positions.

### III. VELOCITY AND FORCE ELLIPSOIDS

For a given posture of a manipulator, there are preferred directions for motion and for force exertion in task coordinates. The optimal directions for accurate control of velocity and force are those that require the maximum velocity and force in actuator coordinates. The velocity and force transmission characteristics of a manipulator at any posture can be represented geometrically as ellipsoids. Consider an \( n \)-degree-of-freedom manipulator with joint coordinates \( \theta_i, i=1,2,.........n \), and a task described by \( m \) task coordinates \( x_j, j=1,2,.....m \) with \( m \leq n \). Let the kinematic transformation from joint space be given by

\[
x = x(\theta)
\]

Where \( \theta=[\theta_1, \theta_2,..... \theta_n]^T \) and \( x=[x_1,x_2,.....x_n]^T \) are the joint and task coordinate vectors. Differentiating with respect to time, we obtain

\[
\dot{x} = J(\theta)\dot{\theta}
\]

Where \( J \) is the \( m \times n \) Jacobian matrix, with elements given by \( J_{ij} = \partial x_i/ \partial \theta_j \). From above equation we see that Jacobian is simply a linear transformation that maps the joint velocity in \( \mathbb{R}^n \) into a task velocity in \( \mathbb{R}^m \). The unit sphere in \( \mathbb{R}^n \) defined by

\[
||\dot{\theta}||^2 = \dot{\theta}_1^2 + \dot{\theta}_2^2 + \cdots + \dot{\theta}_n^2 \leq 1
\]
Is mapped into an ellipsoid in Rm defined by
\[ \dot{x}^T (J J^T)^{-1} \dot{x} \leq 1 \]

Yoshikawa (1985) called this the “manipulability ellipsoid” and proposed that kinematic redundancy should be used to maximize the volume of this ellipsoid. This is an effective way of avoiding singularities. The principal axes of the velocity ellipsoid coincide with Eigen vectors of \((J J^T)^{-1}\), and the length of principal axis is equal to the reciprocal of the square root of the corresponding Eigen value.

**Figure 1. Velocity ellipsoid**

Analogous to the velocity ellipsoid we can also define the force ellipsoid for describing the force transmission characteristics of a manipulator at a given posture. Forces in joint space and task space are mapped via the same jacobian through the relation

\[ \tau = J^T f \]

Where the \( f \) is force vector in task space and \( \tau \) is the joint torque vector. This relation can be derived from equation 1 by equating mechanical power in joint space with that in task space. That is,

\[ \dot{\theta}^T \tau = x^T f = \dot{x}^T J f^T \]

Using equation 3 we obtain

\[ \tau^T \tau = f^T (J J^T) f \leq 1 \]

Hence, the set of achievable force in Rm subject to the constraint \( ||\tau||^2 \leq 1 \) is the ellipsoid defined by

\[ f^T (J J^T) f \leq 1 \]

This is the force ellipsoid. Analogous to the velocity ellipsoid, the principal axes of the velocity ellipsoid coincide with Eigen vectors of \((J J^T)^{-1}\), and the length of principal axis is equal to the reciprocal of the square root of the corresponding Eigen value.
IV. UNIVERSITY OF MARYLAND SPATIAL PARALLEL MANIPULATOR

![Kinematic model of University of Maryland parallel manipulator](image)

The inverse kinematics problem involves as shown in the Fig.2 a reference coordinate system $(x, y, z)$ is attached to the center O of the fixed platform, with its x and y axes lie on the fixed plane and the z-axis points up vertically. Another coordinate system $(x_i, y_i, z_i)$ is attached to the fixed base at point $A_i$, such that the $x_i$-axis is in line with the extended line of the $y_i$-axis is directed along the revolute joint axis at $A_i$, and the $z_i$-axis is parallel to the z-axis. The angle $\phi_i$ is measured from the x-axis to the $x_i$-axis and is a constant parameter of the manipulator design. Fig 4.2 defines the joint angles associated with the $i$th limb, wherein $p$ is the position vector of the centroid of the moving platform, $\theta_i$ is measured from the $x_i$ axis to, $\theta_{z_i}$ is defined from the extended line of to the line defined by the intersection of the plane of the parallelogram and the $x_i$ - $z_i$ plane and $\theta_{3_i}$ is measured from the $y_i$ direction to . Overall, there are nine joint angles, $\theta_{1i}$, $\theta_{2i}$, and $\theta_{3i}$ for $i=1, 2, and 3$ associated with the manipulator.

V. INVERSE KINEMATIC ANALYSIS OF SPATIAL PARALLEL MANIPULATOR

For inverse kinematics, the position vector $P$ of the moving platform is given and the problem is to find the joint angles $\theta_{1i}$, $\theta_{12}$, and $\theta_{13}$ required to bring the moving platform to the desired position. An intuitive approach for the solution is to consider the problem geometrically. If the position of $P$ is given, then the position of $C_i$ also known.

From the loop-closer equation of each limb the following expression is obtained

$$
\begin{bmatrix}
ac \theta_i + bs \theta_{cn} (	heta_{ai} + \theta_{b1}) \\
bc \theta_{ni} \\
as \theta_{ni} + bs \theta_{nc} (\theta_{a1} + \theta_{b1})
\end{bmatrix} =
\begin{bmatrix}
C_{x_i} \\
C_{y_i} \\
C_{z_i}
\end{bmatrix}
$$

(1)

From Equation (1)

$$
\theta_{3i} = \cos^{-1}(C_{yi}/b)
$$

(2)

With $\theta_{3i}$ determined, an equation with $\theta_{2i}$ as the only unknown is generated by summing the squares of $c_{xi}$, $c_{yi}$, and $c_{zi}$

$$2\text{abs } \theta_{3i} \text{c}\theta_{2i} + a^2 + b^2 = c_{xi}^2 + c_{yi}^2 + c_{zi}^2$$  (3)

Hence

$$\theta_{2i} = \cos^{-1} k.$$  (4)

Where

$$k = \left( c_{xi}^2 + c_{yi}^2 + c_{zi}^2 \right) / \left( 2\text{abs}\theta_{3i} \right)$$  (5)

Therefore, corresponding to each solution of $\theta_{3i}$, yields two solutions of $\theta_{2i}$. This results in four solution sets for $\theta_{2i}$ and $\theta_{3i}$. Corresponding to each solution set of $\theta_{2i}$ and $\theta_{3i}$, yields a unique solution for $\theta_{1i}$.

VI. JACOBIAN ANALYSIS OF PARALLEL MANIPULATORS

For the manipulator, the input vector is $\mathbf{v} = [\mathbf{v}_{\text{p},x}, \mathbf{v}_{\text{p},y}, \mathbf{v}_{\text{p},z}]^T$. All other joint rates are passive variables.

$$\mathbf{J} \mathbf{v}_{\text{p}} = \mathbf{J}\mathbf{q},$$

Where

$$\mathbf{J}_x = \begin{bmatrix} j_{1x} & j_{1y} & j_{1z} \\ j_{2x} & j_{2y} & j_{2z} \\ j_{3x} & j_{3y} & j_{3z} \end{bmatrix}$$

$$\mathbf{J}_q = a \begin{bmatrix} s\theta_{21}s\theta_{31} & 0 & 0 \\ 0 & s\theta_{22}s\theta_{32} & 0 \\ 0 & 0 & s\theta_{23}s\theta_{33} \end{bmatrix}$$

Jacobian Matrix $\mathbf{J} = \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{bmatrix}$

$$J_{11} = \frac{c(\theta_{11} + \theta_{21})s\theta_{31}s\theta_{01} - c\theta_{31}s\theta_{01}}{s\theta_{21}s\theta_{01}}$$

$$J_{12} = \frac{c(\theta_{11} + \theta_{21})s\theta_{31}s\theta_{01} + c\theta_{31}s\theta_{01}}{s\theta_{21}s\theta_{01}}$$

$$J_{13} = \frac{s(\theta_{11} + \theta_{21})s\theta_{31}s\theta_{01}}{s\theta_{21}s\theta_{01}}$$

$$J_{21} = \frac{c(\theta_{12} + \theta_{22})s\theta_{32}s\theta_{02} - c\theta_{32}s\theta_{02}}{s\theta_{22}s\theta_{02}}$$

$$J_{22} = \frac{c(\theta_{12} + \theta_{22})s\theta_{32}s\theta_{02} + c\theta_{32}s\theta_{02}}{s\theta_{22}s\theta_{02}}$$

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VII. RESULTS

A MATLAB code is generated for the inverse kinematic analysis, Jacobian matrix and Manipulability Index of the University of Maryland manipulator. After verifying the results of various kinematic structures and different configurations in each structure, the optimal structure is identified based on manipulability index.

\[ J_{23} = \frac{s(0.12 + 0.22)s0.32}{s0.22s0.32} \]
\[ J_{31} = \frac{c(0.13 + 0.23)s0.33c0.3 - c0.33s0.3}{s0.21s0.33} \]
\[ J_{32} = \frac{c(0.13 + 0.23)s0.33c0.3 + c0.33s0.3}{s0.23s0.33} \]
\[ J_{33} = \frac{s(0.13 + 0.23)s0.33}{s0.23s0.33} \]
VIII. CONCLUSION

The University of Maryland manipulator has been analyzed for its velocity transmission capabilities, using the Yoshikawa Manipulability index concept. After verifying the results of different structures based on manipulability index the optimal structures are identified.

REFERENCES