SIMPLIFICATION OF ECONOMIC PRODUCTION QUANTITY MODEL BY EQUIVALENT HOLDING COST

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ABSTRACTS

In traditional Economic Order Quantity (EOQ) model, replenishment is in one lot however in Economic Production Quantity (EPQ) model the supply of order quantity is at uniform rate. The EPQ model is little more complex and formulae for calculation of EPS, Total cost, and Holding cost are more complex and difficult (as compare to EOQ). We use the concept of Equivalent Holding Cost (EHC) or Equivalent Carrying Cost. With this concept EPQ model is no different from EOQ model in terms of formulae. All formulae of EOQ model could be used for EPQ model only putting Equivalent Holding cost in place of holding cost! It has really simplified the EPQ model.

Key Words: ELS, EPQ, Holding, Equivalent holding cost, EHS

1. INTRODUCTION

Economic Order Quantity (EOQ) model is illustrated in almost all the literature. Some authors uses EPQ (Economic Lot Size) while many authors prefer EPQ (Economic
Production Quantity) for the model where in production of lot is commences as soon as stock position reaches zero inventory and items are also supplied at uniform demand rate. Naturally, production rate should be much more than demand rate.

The assumptions for ERL model are given in (Vora, 2001). Gupta (2006) have noted almost all the model including EPQ. Some authors prefer carrying cost (Gupta, 2006; Sharma, 2007; Sharma S.D., 2005; …) while some authors use holding cost (Vora, 2001; Anderson, 2006; Dilworth, 1996; Gupta, 2006). Holding cost is more meaningful in inventory model context and hence should be preferred to carrying cost.

Inventory is waste and should be minimized, as it could not be eliminated. Inventory models mainly:
1) EOQ,
2) EPQ,
3) EOQ with planned shortages and
4) EPQ with planned shortages;
should be used in order to minimize the total inventory cost. However last three models are more complex, requires very complicated formulae; and are not included in much literature. Gupta (2006), Pannerselvam (2006), Sharma (2007), Sharma S (2005), have included model (3) and (4). Most of the western literature do not include model (3) and (4) (Dilworth, 1996; Narsimhan, 2009; Taha, 2006; …). This is all do to complicated derivations and complex formulae involved in these two models.

We have developed the concept of Equivalent Holding Cost for model (2) in this manuscript. We developed the concept of EHC also for model (3) (Kharde, Vikhe Patil and Nandurkar, 2012); and for model (4) (Kharde, Vikhe Patil and Nandurkar, 2011). As a result, Inventory models are very much simplified by usage of Equivalent Holding cost (EHC) and surprisingly complex models have been now very simple to use and practice! Because just use EOQ formulae- instead of Holding cost, use Equivalent Holding Cost.

2. NOTATION

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
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<tbody>
<tr>
<td>Q</td>
<td>The production lot quantity</td>
</tr>
<tr>
<td>Q*</td>
<td>Economic Production Quantity (EPQ) or Economic Lot Size (ELS)</td>
</tr>
<tr>
<td>O</td>
<td>Set up cost per set up</td>
</tr>
<tr>
<td>H</td>
<td>Holding Cost/unit/year; or Carrying cost/unit/year</td>
</tr>
<tr>
<td>T(Q)</td>
<td>Total annual inventory cost</td>
</tr>
<tr>
<td>O(Q)</td>
<td>Total annual set up cost</td>
</tr>
<tr>
<td>H(Q)</td>
<td>Total annual holding cost</td>
</tr>
<tr>
<td>p</td>
<td>Uniform production rate; units/time</td>
</tr>
<tr>
<td>d</td>
<td>Uniform demand rate; units/time</td>
</tr>
<tr>
<td>t_0</td>
<td>production run time</td>
</tr>
<tr>
<td>T</td>
<td>cycle time; or time between two consecutive set-ups</td>
</tr>
<tr>
<td>LT</td>
<td>Lead time</td>
</tr>
<tr>
<td>R</td>
<td>Re-order-level</td>
</tr>
<tr>
<td>K_p</td>
<td>Factor for equivalent holding cost for uniform production, ( \left(1 - \frac{d}{p}\right) )</td>
</tr>
<tr>
<td>H_{ep}</td>
<td>Equivalent Holding Cost (EHC) for production model</td>
</tr>
<tr>
<td>M</td>
<td>Maximum Inventory level</td>
</tr>
<tr>
<td>D</td>
<td>Total annual demand; Demand/year</td>
</tr>
</tbody>
</table>
3. **ECONOMIC PRODUCTION QUANTITY (EPQ) MODEL**

Certain assumptions are made in this model like Wilson's formulation [12]:
1. The demand for the item is certain, constant and continuous
2. Lead time is fixed
3. Holding cost (H) per unit per unit time is constant and does not change for different order quantity
4. Ordering cost (O) per order is constant and does not vary with number of orders
5. Purchase price of the item is constant and does not change within the period of planning. No discount is applicable
6. The Production of the item start immediately as stock level reaches zero at constant production rate (p) and same is replenish at demand rate (d)
7. The demand for the item is uniform and constant (d)
8. No stock outs are permitted.
9. Uniform production rate 'p' is greater than uniform demand rate 'd'.

Production starts at origin, produced at rate 'p' and supplied at rate 'd'. The net inventory built up rate is 'p-d' (figure 1). This continues till production quantity is manufactured (Q*). At this point production stops and only demand is at rate of 'd'. Production order is put when stock level reaches the reorder point. The new lot production starts when stock level reaches zero.

4. **EQUIVALENT HOLDING COST (EHC)**

During production period 't_p' inventory built up rate is 'p - d'
Maximum Inventory level, M
\[
M = t_p (p - d) \tag{1}
\]
As minimum inventory is zero and \( t_p = \frac{Q}{p} \) we have
Average inventory = \( t_p \left( \frac{p - d}{2} \right) \)

\[
= \left( \frac{Q}{p} \right) \left( \frac{p - d}{2} \right)
\]

In this model average inventory is different by factor \( (p – d)/p \). In EOQ model average inventory is only \( Q/2 \). This factor is a factor for change in the inventory level of ERL model from EOQ model.

Annual Holding cost

\[
H(Q) = \left( \frac{Q}{p} \right) \left( \frac{p - d}{2} \right) H
\]

\[
H(Q) = \left( \frac{Q}{2} \right) \left( \frac{p - d}{p} \right) H \tag{2}
\]

Factor for uniform production

We define factor \( K_p \) as Factor for Uniform Production

\[
K_p = \left( \frac{p - d}{p} \right) \tag{3}
\]

Then from equation (2) and (3)

\[
H(Q) = \left( \frac{Q}{2} \right) (K_p) H \tag{4}
\]

In EOQ model Annual inventory cost is

\[
H(Q) = \left( \frac{Q}{2} \right) H
\]
Here $K_p^*H$ is just an equivalent holding cost in ERL model when compared with EOQ Equivalent Holding Cost (EHC)

Defining Equivalent Holding Cost (EHC) in ERL model

The factor is denoted by $H_{ep}$ i.e. equivalent holding cost for production or ERL model.

$$H_{ep} = K_p^*H$$  \hspace{1cm} (5)

Then from equation (4) and (5), the Annual Holding Cost for ERL model is

$$H(Q) = \left(\frac{Q}{2}\right)H_{ep}$$  \hspace{1cm} (6)

5. **EPQ MODEL DERIVATION WITH EHC**

**Annual set-up cost**

$$O(Q) = \left(\text{number of setups per year}\right)\left(\text{setup cost per setup}\right)$$

$$O(Q) = \left(\frac{D}{Q}\right)O$$  \hspace{1cm} (7)

**Total annual inventory cost**

From equation (6) and (7), we get

$$T(Q) = H(Q) + O(Q)$$

$$T(Q) = \left(\frac{Q}{2}\right)H_{ep} + \left(\frac{D}{Q}\right)O$$  \hspace{1cm} (8)

In order to minimize the annual total cost, take derivative w.r.t. $Q$ and equate it to zero.

$$\frac{dT(Q)}{dQ} = \frac{d}{dQ}\left(\left(\frac{Q}{2}\right)H_{ep} + \left(\frac{D}{Q}\right)O\right)$$

$$0 = \left(\frac{1}{2}\right)H_{ep} + \left(-1\right)\left(\frac{D}{Q^2}\right)O$$

$$Q^2 = \frac{2D.O}{H_{ep}}$$

$$Q = \left(\frac{2D.O}{H_{ep}}\right)^{\frac{1}{2}}$$  \hspace{1cm} (9)

This value is optimal, but to confirm for minimum, second derivative must be positive.

$$\frac{d^2T(Q)}{dQ^2} = \frac{d}{dQ}\left(\left(\frac{1}{2}\right)H_{ep} + \left(-1\right)\left(\frac{D}{Q^2}\right)O\right)$$

$$= 0 - \left(-2\right)\left(\frac{D}{Q^3}\right)O$$

$$= 2 \left(\frac{D}{Q^3}\right)O$$

As all quantities on RHS are positive (O, D and Q); RHS is greater than zero

$$= 2 \left(\frac{D}{Q^3}\right)O > 0$$

Hence the quantity (Q) found is the optimum quantity to give minimum total cost point; denoting as $Q^*$

Hence for ERN or EPQ model from equation (9)

$$Q^* = \left(\frac{2D.O}{H_{ep}}\right)^{\frac{1}{2}}$$  \hspace{1cm} (10)

For classical EOQ model this formula is:
\[ q^* = \left( \frac{2D \cdot O}{H} \right)^{0.5} \]

It is confirmed that only EHC should be used instead of holding cost and formula remains unchanged.

**Total Optimum Annual Inventory cost**

Taking equation (8) and putting the value of \( q^* \)

\[
T(Q) = \left( \frac{Q}{H} \right) H_{sp} + \left( \frac{D}{Q} \right) O
\]

\[
= \left( \frac{H_{sp}}{2} \right) \left( \frac{2D \cdot O}{H_{sp}} \right)^{0.5} + \frac{D \cdot O}{\left( \frac{2D \cdot O}{H_{sp}} \right)^{0.5}}
\]

\[
T(Q) = \left( \frac{2D \cdot O \cdot H_{sp}}{O} \right)^{0.5} \quad (11)
\]

So all formulae for ERL model are like EOQ model.

<table>
<thead>
<tr>
<th>Table 1: ERL Model Formulae</th>
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<tbody>
<tr>
<td>1</td>
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<tr>
<td>2</td>
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<td>4</td>
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<td>6</td>
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<tr>
<td>7</td>
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<td>8</td>
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</tbody>
</table>

Table 2 shows Formulae comparison…

**Table 2 - Formulae Comparison**

<table>
<thead>
<tr>
<th>Item</th>
<th>EOQ</th>
<th>EPQ</th>
<th>EPQ with EHC</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_p )</td>
<td>-</td>
<td>-</td>
<td>( (p - d)/p )</td>
</tr>
<tr>
<td>( H_{sp} )</td>
<td>H</td>
<td>H</td>
<td>( K_p \cdot H )</td>
</tr>
<tr>
<td>( Q^* )</td>
<td>( \left( \frac{2D \cdot O}{H} \right)^{0.5} )</td>
<td>( \left( \frac{2D \cdot O}{H} \cdot \frac{p - d}{d} \right)^{0.5} )</td>
<td>( \left( \frac{2D \cdot O}{H_{sp}} \right)^{0.5} )</td>
</tr>
<tr>
<td>( T(Q^*) )</td>
<td>( (2D \cdot O)^{0.5} )</td>
<td>( \left( 2D \cdot O \left( \frac{p - d}{d} \right) \right)^{0.5} )</td>
<td>( (2D \cdot O)^{0.5} )</td>
</tr>
<tr>
<td>( H(Q) )</td>
<td>( \left( \frac{Q^*}{2} \right) H )</td>
<td>( \left( \frac{Q}{2} \right) \left( \frac{p - d}{p} \right) H )</td>
<td>( \left( \frac{Q^*}{2} \right) H_{sp} )</td>
</tr>
</tbody>
</table>
Illustration with Examples

**Example: 1**

Test problem (Vora, 2001)

<table>
<thead>
<tr>
<th>D</th>
<th>2,500 units/year</th>
</tr>
</thead>
<tbody>
<tr>
<td>O</td>
<td>250 $/set-up</td>
</tr>
<tr>
<td>H</td>
<td>4 $/unit/year</td>
</tr>
<tr>
<td>p</td>
<td>4,800 units/year</td>
</tr>
<tr>
<td>d</td>
<td>2,500 units/year</td>
</tr>
</tbody>
</table>

First calculate *Factor for equivalent holding cost* for uniform production, \( K_p \) and *EHC*

\[
K_p = \frac{(p-d)}{p} = \frac{(4800-2500)}{4800} = 0.4792
\]

\[
H_{ep} = (K_p)(H) = (0.4792)(4) = 1.9167 \text{ $/unit/year}
\]

\[
Q^* = \left[ \frac{2(D)(O)}{H_{ep}} \right]^{0.5} = \left[ \frac{2(2500)(250)}{1.9167} \right]^{0.5} = 807.57 \text{ units per production lot}
\]

\[
T(Q^*) = \left[ \frac{2(D)(O)(H_{ep})}{Q^*} \right]^{0.5} = \left[ \frac{2(2500)(250)(1.9167)}{807.57} \right]^{0.5} = 1547.84 \text{ $/year}
\]

\[
H(Q^*) = \frac{(Q^*/2)(H_{ep})}{2} = \frac{(808/2)(1.9167)}{2} = 773.92 \text{ $/year}
\]

\[
O(Q^*) = \frac{(D)(O)}{Q^*} = \frac{(2500)(250)}{808} = 773.92 \text{ $/year}
\]

\[
H(Q^*) + O(Q^*) = T(Q^*) \text{ verified}
\]

\[
t_p = \frac{(Q^*)}{p} = \frac{808}{4800} = 0.1683 \text{ year} = 51 \text{ days; assuming 300 WD}
\]

\[
T = \frac{(Q^*)}{d} = \frac{808}{2500} = 0.3232 \text{ year} = 97 \text{ days; assuming 300 WD}
\]
Example: 2
Test problem (Vora, 2001) is illustrated here

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<table>
<thead>
<tr>
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<tbody>
<tr>
<td>D</td>
<td>60,000 units/year</td>
</tr>
<tr>
<td>O</td>
<td>300 $/set-up</td>
</tr>
<tr>
<td>H</td>
<td>10 $/unit/year</td>
</tr>
<tr>
<td>p</td>
<td>500 units/day</td>
</tr>
<tr>
<td>d</td>
<td>200 units/day</td>
</tr>
</tbody>
</table>

1. \[ K_p = \frac{(p - d)}{p} \]
   \[ = \frac{(500-200)}{500} = 0.6 \]

2. **Equivalent Holding Cost**
   \[ H_{ep} = (K_p)(H) \]
   \[ = (0.6)(10) = 6 \]$/unit/year

3. **Economic Lot Size**
   \[ Q^* = \left[ \frac{2(D)(O)}{H_{ep}} \right]^{0.5} \]
   \[ = \left[ \frac{2(60000)(300)}{6} \right]^{0.5} = 2449.48 = 2450 \] units per lot

4. **Total Optimum Annual Inventory cost**
   \[ T(Q^*) = \left[ \frac{2(D)(O)(H_{ep})}{2} \right]^{0.5} \]
   \[ = \left[ \frac{2(60000)(300)(6)}{6} \right]^{0.5} = 14696.94 \]$ /year

5. **Total Annual Holding cost**
   \[ H(Q^*) = \frac{(Q^*)}{2} (H_{ep}) \]
   \[ = \frac{(2449.48)}{2}(6) = 7348.47 \]$/year

6. **Total Annual Set-up cost**
   \[ O(Q^*) = \frac{D(O)}{Q^*} \]
   \[ = \frac{(60000)(300)}{2449.48} = 7348.47 \]$/year
   \[ H(Q^*) + O(Q^*) = T(Q^*) \] verified

7. **Production Period per cycle**
   \[ t_p = \frac{(Q^*)}{p} \]
   \[ = \frac{2450}{500} = 5 \] days

8. **Time between two set-ups, cycle time**
   \[ T = \frac{(Q^*)}{d} \]
   \[ = \frac{2450}{200} = 12.25 \] days = 13 days;

6. **CONCLUSIONS**

Factor for equivalent holding cost for uniform production is very simple to calculate. The Equivalent Holding Cost (EHC) for EPQ model can be calculated from this factor. The concept of EHC has simplified the EPQ or EPQ model to EOQ model. All formulae for EOQ model can be used for EPQ with 'EHC' in place of 'H'. Unbelievable simplification is resulted from this concept of EHC. No need henceforth to use the complicated formulae of old EPQ model as given in all literature. The notations used in this paper are simple and could be used as standard.
REFERENCES