ANALYSIS AND COMPARISON OF VEHICLE DYNAMIC SYSTEM WITH NONLINEAR PARAMETERS SUBJECTED TO ACTUAL RANDOM ROAD EXCITATIONS

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ABSTRACT

All system exhibit nonlinear characteristics in practice. The importance of effects depend upon the degree of nonlinearity and so the effect on the response. Dynamic system constitutes mass, spring and viscous damper. In this paper, nonlinearity in mass, spring and viscous damper are considered and compared for their individual and relative significance. Also, it is studied how nonlinearity affects the response compared to linear system. The nonlinearity in mass arises as when car moves with velocity, which is due to change in mass density of fluid around it. Nonlinearity in spring stiffness and damping coefficient obtained from actually measured data of a car and nonlinearity in stiffness and damping (rolling dynamic stiffness and damping coefficient) of tire are applied to simulate the nonlinearities in two degree freedom quarter car vehicle dynamic system. The theories of non-linear dynamics are applied to study non-linear model and to reveal its non-linear vibration characteristics. Thus this paper deals with comparison between simulation results obtained for passive and semi-active linear systems with nonlinear mass, spring and damper controller. The excitation is taken as actual random road excitation to achieve improved performance. Thus, the emphasis is to study the nonlinearities in mass, spring and damper for passive suspension system performance and compare the reactive significance.

Keywords: Effective Mass, Non-linear Suspension, Quarter Car Model, Random Road Surface, Vehicle Dynamic System,
I. INTRODUCTION

In order to enhance ride safety and ride comfort, it is essential to estimate vehicle dynamic response to design automotive systems. These compulsions have motivated automotive industries to use active and semi-active suspension in middle top range due to their effectiveness [1]. Many analytical and experimental studies on active and semi-active suspensions have been performed to improve ride quality and handling performance [2]. Though passenger cars are complex and multibody systems, its essential dynamics can be represented by a quarter car model [3]. The stiffness and damping parameters are fixed and effective over a certain range of frequencies in passive suspension system. In order to overcome this problem, the active suspension system having capability of adopting changing road conditions with the use of an actuator have been considered. [4]

Increased cost, complexity, need for an external energy source and difficulty in control hardware implementation, active suspension system led to development of semi-active suspension systems, which combine the advantages of both passive and active suspension systems [5]. Semi-active suspensions have been considered by number of authors for vehicular vibration control applications [6-9].

In view of handling characteristics, the vehicle is considered to be within linear region when its lateral acceleration is less than 0.4 g, which represents the normal driving condition unless it experiences severe cornering. But in view of ride related movement, the linear region is relatively small, for example, the permissible rattle space of a passenger car is less than 150 mm due to the limitation of the body movement point. But the suspension load versus wheel displacement curve shows that the linearity is less than 50 mm mainly during rebound movement. Therefore it is very important to include nonlinearities in order to account for a more realistic operation of the vehicle [10].

In practice, the mass of vehicle does not remain constant when it is in motion, the amount by which it exceeds depends upon its volume/shape and the mass density of the fluid around it [11]. The vehicle suspensions are not really linear, due to friction (stiction) in the struts and bushings, or the interleaf friction in a leaf spring. Rather than a simple linear relationship between force and displacement, the suspension exhibits polynomial behavior. Similarly due to change in physical parameters and chemical properties of the damper parts, the relationship between damping force and velocity is also not linear but it also exhibits polynomial behavior. The tire is represented as a simple spring, and damper is often included to represent the small amount of damping inherent to the visco-elastic nature of the tire.

Many workers have carried out dynamic response and dynamics control with linear vehicle models. However, due to elastic components with non-linear characteristics, the system behaves like non-linear. In this study, a realistic non-linear suspension model which comprises of non-linear mass, spring and damper is considered[12]. From comparisons of results of analysis of passive suspension, it is shown how the nonlinearities in mass, suspension spring, damper and tire affect the response and how worth it is to consider the effect of nonlinearities in the system parameters. Thus, the objective of this paper is to do comparative study of nonlinearities in mass, spring and damper for passive suspension system and determine the relative significance.
II. PROBLEM FORMULATION

In order to describe the vehicle dynamics of road vehicle, two degree freedom linear passive quarter car model shown in Fig. 1 is used as it is simple and captures all important characteristics of full car model.

![Linear Passive Quarter Car Model](image)

The equations of motion for bouncing motion of Linear Two DoF vehicle passive model are

\[ m_s \ddot{x}_s = -\left[k_s (x_s - x_u) + c_s (\dot{x}_s - \dot{x}_u)\right] \]

\[ m_u \ddot{x}_u = \left[-k_s (x_s - x_u) - c_s (\dot{x}_s - \dot{x}_u) - k_t (q - x_u) - c_t (\dot{q} - \dot{x}_u)\right] \]

By considering nonlinearities in mass, suspension spring, damper and tire, the equations of motion for body bouncing motion for Nonlinear Two DoF vehicle passive model are

\[ f_{ls} = -f_s - f_d - m_s g \]

\[ f_{lu} = +f_s + f_{st} + f_d + f_{dt} - m_u g \]

Where,

\( f_{ls} \) - Inertia force due to sprung mass,

\( f_s \) – Suspension spring force,

\( f_d \) – Suspension damper force,

\( m_s g \) - Sprung Weight,

\( f_{lu} \) - Inertia force due to unsprung mass,

\( f_{st} \) - Tire Spring force,

\( f_{dt} \) – Tire damper force and
$m_{ug}$ - Unsprung Weight.

In the linear model, these connecting forces were described as linear functions of the states of the system $f_s = kx$, $f_d = cx$, $f_s = m_s \ddot{x_s}$ and $f_{is} = m_i \ddot{x_i}$. The values of $k$ and $c$ were obtained from measured data on SPMD. Tire stiffness ($k_t$) and tire damping coefficient ($c_t$) are modeled as non-linear functions with respect to vehicle speed [13].

In this section, the connectivity forces (e.g. inertia force, spring force and damping force) are modeled as non-linear functions. The data measured on SPMD (Suspension Parameter Measurement Device) [14] is used for spring and damper. The vertical tire force becomes zero when the tire loses the contact with the road. To make the suspension model more realistic, this “lift off” is modeled in this study.

**Analysis of TDoF Vehicle Model with Passive Suspension System:**

(a) **Nonlinearity in Sprung Mass.**

The nonlinear effects included in the inertia force are due to continuously changing mass. This change is due to change in mass density of the fluid around the vehicle body. The variation of grew mass of the vehicle body depends upon distance of point of interest from its C.G. for a fluid of constant mass density around it.

Therefore for a spherical object the total effective mass will be $M= m + m'$

Where, $m$ = mass of the spherical object, and $m'$ = Grew mass of spherical object

$$m' = \frac{2R^3\pi}{3} \rho$$

Where, $R$ = Radius of spherical object, and $\rho$ = Mass density of the fluid around a spherical object.

But as vehicle (Hundai Elentra 1992 model car) body is ellipsoidal in shape, the distance $R$ (Effective radius) varies from $a$ (Semi-height of car) to $b$ (Semi-width of car) and $b$ (Semi-width of car) to $c$ (Semi-length of car).

Thus, the non-linear inertia force due to nonlinear sprung mass is modeled as shown in Fig. 2.
Fig 2: Nonlinear (Effective) sprung mass property of ellipsoidal car body.

The nonlinear inertia force due to sprung mass \( f_{ls} \) is modeled as third order polynomial function as

\[
f_{ls} = m_{S2}r^2 + m_{S1}r + m_{S0}
\]

Where, \( m_{S1}, m_{S2}, m_{S3} \) are the constants and are determined as

\[
m_{S2} = 10.11, m_{S1} = -13.71 \text{ and } m_{S0} = 220.2
\]

Therefore, the equations of motion for nonlinear mass passive suspension system are:

\[
\ddot{x}_s = -\frac{1}{(m_{S2}r^2 + m_{S1}r + m_{S0})} \left[ k_s (x_s - x_u) + c_s (\dot{x}_s - \dot{x}_u) \right]
\]

\[
\ddot{x}_u = -\frac{1}{m_u} \left[ -k_s (x_s - x_u) + c_s (\dot{x}_s - \dot{x}_u) \right] - \left[ k_t (q - x_u) + c_t (\dot{q} - \dot{x}_u) \right]
\]

The parameters of Hyundai Elantra 1992 model considered for the analyses are given in Table 1.

<table>
<thead>
<tr>
<th>Sprung Mass (m_s)</th>
<th>214.65 Kg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unsprung Mass (m_u)</td>
<td>21.46 Kg</td>
</tr>
<tr>
<td>Suspension Stiffness (k_s)</td>
<td>12394 N/m</td>
</tr>
<tr>
<td>Passive Suspension Damping Coefficient (c_s)</td>
<td>1385.4 N-sec/m</td>
</tr>
<tr>
<td>Semi-active Suspension Damping Coefficient (c_{ss})</td>
<td>35441 N-sec/m</td>
</tr>
<tr>
<td>Tyre Stiffness (k_t)</td>
<td>123940 N/m</td>
</tr>
<tr>
<td>Tyre Damping Coefficient (c_t)</td>
<td>138.54 N-sec/m</td>
</tr>
</tbody>
</table>

The sprung mass acceleration for linear passive and nonlinear (effective) sprung mass passive suspension system for random road excitation (q) is as shown in Fig.3.
Fig. 3: Sprung mass acceleration for linear passive and nonlinear sprung mass passive controller

These results are tabulated in Table 2

<table>
<thead>
<tr>
<th>Input</th>
<th>Controller</th>
<th>Sprung Mass Acceleration Range in m/s²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random Road</td>
<td>Linear Passive</td>
<td>-5.00 to 4.49</td>
</tr>
<tr>
<td>Nonlinear Sprung</td>
<td>Nonlinear Sprung Mass Passive</td>
<td>-4.87 to 4.39</td>
</tr>
</tbody>
</table>

Result - It can be observed that due to sprung mass nonlinearity, the response deviates from linear system by 2.22% at upper side while 2.6% at lower side.

(b) **Nonlinearity in Unsprung Mass.**

Nonlinear (effective) unsprung mass is modeled as shown in Fig 4
The nonlinear inertia force due to sprung mass $f_{iu}$ is modeled as third order polynomial function as

$$f_{iu} = m_{u2}r^2 + m_{u1}r + m_{u0}$$

Where,

$m_{u2} = 1.123$, $m_{u1} = -1.523$ and $m_{u0} = 22.09$

Therefore, the equations of motion for nonlinear mass passive suspension system are:

$$\ddot{x}_s = -\frac{1}{(m_s)}[k_s(x_s - x_u) + c_s(\dot{x}_s - \dot{x}_u)]$$

$$\ddot{x}_u = -\frac{1}{(m_{u2}r^2 + m_{u1}r + m_{u0})}\left\{[k_s(x_s - x_u) + c_s(\dot{x}_s - \dot{x}_u)] - [k_i(q - x_u) + c_i(q - \dot{x}_u)]\right\}$$

The sprung mass acceleration for linear passive and nonlinear unsprung mass passive suspension system for random road excitation (q) are as shown in Fig.5

![Sprung mass acceleration for linear passive and nonlinear Unsprung mass passive controller](image)

Fig.5: Sprung mass acceleration for linear passive and nonlinear unsprung mass passive controller

These results are tabulated in Table 3

<table>
<thead>
<tr>
<th>Input</th>
<th>Controller</th>
<th>Sprung Mass Acceleration Range in m/s²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random Road Acceleration</td>
<td>Linear Passive</td>
<td>-5.00 to 4.49</td>
</tr>
<tr>
<td></td>
<td>Non linear Unsprung Mass Passive</td>
<td>-4.99 to 4.48</td>
</tr>
</tbody>
</table>
Result- It can be observed that due to unsprung mass nonlinearity, the response deviates from linear system by 0.22% at upper side while 0.20% at lower side.

(c) Nonlinearity in suspension spring.
The non-linear effects included in the spring force $f_s$ are due to two parts. One is bump stop, which restricts the wheel travel within the given range and prevents the tire from containing the vehicle body. And the other is strut bushing which connects the strut with the body structure and reduces the harshness from the road input. These non-linear effects can be included in spring force $f_s$ with non-linear characteristic versus suspension rattle space $(x_s - x_u)$ from measured data (SPMD) shown in Fig. 6.

Fig. 6. Modeling of nonlinear spring force [wheel stroke (m) Vs suspension force (N)]
The spring force $f_s$ is modeled as third order polynomial function as,

$$f_s = k_0 + k_1 x + k_2 x^2 + k_3 x^3$$

Where the co-efficients are obtained by fitting the experimental data, which resulted in $k_3 = 3170400 \text{ N/m}^3$, $k_2 = -73696 \text{ N/m}^2$, $k_1 = 12394 \text{ N/m}$ and $k_0 = -2316.4 \text{ N}$ (The SPMD data from the 1992 model Hyundai Elantra front suspension were used).

Therefore, the equations of motion for nonlinear suspension spring passive system are:

$$\ddot{x}_s = \frac{1}{m_s} \left[ k_0 + k_1 (x_s - x_u) + k_2 (x_s - x_u)^2 + k_3 (x_s - x_u)^3 + c_s (\dot{x}_s - \dot{x}_u) + m_s g \right]$$

$$\ddot{x}_u = -\frac{1}{m_u} \left\{ k_0 + k_1 (x_u - x_s) + k_2 (x_u - x_s)^2 + k_3 (x_u - x_s)^3 + c_s (\dot{x}_u - \dot{x}_s) - k_s (q - x_u) + c_f (\dot{q} - \dot{x}_u) \right\}$$

The sprung mass acceleration for linear passive and nonlinear suspension spring passive suspension system for random road excitation (q) are as shown in Fig. 7.
Fig. 7: Sprung mass acceleration for linear passive and nonlinear suspension spring passive controller

The results obtained are tabulated in Table 4

<table>
<thead>
<tr>
<th>Input</th>
<th>Controller / Parameter</th>
<th>Sprung Mass Acceleration Range in m/s²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random Road Acceleration</td>
<td>Linear Passive</td>
<td>-5.00 to 4.49</td>
</tr>
<tr>
<td></td>
<td>Nonlinear suspension spring Passive</td>
<td>-8.16 to 6.46</td>
</tr>
</tbody>
</table>

Result: It can be observed that due to suspension spring nonlinearity, the response deviates from linear system by 44.09% at upper side while 63.20% at lower side %

(d) **Nonlinearity in tire spring.**

The non-linear effects included in the tire stiffness depend on the speed of the vehicle, inflation pressure and ply type of wheel. There are three types of tire vertical stiffnesses defined static, nonrolling dynamic and rolling dynamic stiffness. These non-linear effects can be included in spring force $f_t$ with non-linear stiffness characteristics with respect to speed.

The tire stiffness is modeled as shown in Fig.8.
Therefore, tire spring force $f_u$ is modeled as third order polynomial function as,

$$f_u = (k_0 + k_1v + k_2v^2 + k_3v^3)(q - x_u)$$

When

$$ (q - x_u) > 0$$

And

$$f_u = 0$$

When

$$ (q - x_u) \leq 0$$

The co-efficients are obtained by fitting the experimental data, which resulted in

- $k_3 = -19.12 \text{ N-s}^2/\text{m}^4$
- $k_2 = 520.3612 \text{ N-s}^2/\text{m}^3$
- $k_1 = -5501.6 \text{ 12 N-s/m}^2$
- $k_0 = 199987.71 \text{N/m}$.

Therefore, the equations of motion for nonlinear tire spring passive system are:

$$\ddot{x}_s = -\frac{1}{(m_s)}[k_s(x_s - x_u) + c_s(\dot{x}_s - \dot{x}_u)]$$

$$\ddot{x}_u = -\frac{1}{(m_u)}\{-[k_s(x_s - x_u) + c_s(\dot{x}_s - \dot{x}_u)] - [(k_0 + k_1v + k_2v^2 + k_3v^3)(q - x_u) + c_I(q - \dot{x}_u)] + m_u g\}$$

The sprung mass acceleration for linear passive and nonlinear tire spring passive suspension system for random road excitation ($q$) are as shown in Fig.9.
The results obtained are tabulated in Table 5

Table 5: Sprung Mass Acceleration for linear passive and nonlinear tire spring passive controller

<table>
<thead>
<tr>
<th>Input</th>
<th>Controller / Parameter</th>
<th>Sprung Mass Acceleration Range in m/s²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random Road Acceleration</td>
<td>Linear Passive</td>
<td>5.00 to 4.49</td>
</tr>
<tr>
<td></td>
<td>Non linear tire spring Passive</td>
<td>-4.88 to 4.32</td>
</tr>
</tbody>
</table>

Result- It can be observed that due to tire spring nonlinearity, the response deviates from linear system by 3.78% at upper side while 2.4% at lower side.

(e) **Nonlinearity in suspension damper.**

Generally, the damping force is asymmetric with respect to speed of the rattle space, damping force during bump is bigger than that during rebound in order to reduce the harshness from the road during bump while dissipating sufficient energy of oscillation during rebound at the same time. Fig.6 shows the measured data for the damping force versus relative velocity of upper and lower struts, shows the asymmetric property.
Fig. 10 Asymmetric damping property of actual suspension system [damper speed (m/s) Vs damping force (kN)]

From the measured data the damping force $f_d$ is modeled as second order polynomial function as,

$$f_d = c_1 \Delta \dot{x} + c_2 \Delta \ddot{x}^2$$

Where the co-efficients are obtained from fitting the experimental data, which resulted in

$c_2 = 524.28$ N-s$^2$/m$^2$ and $c_1 = 1385.4$ N-s/m.

Therefore, the equations of motion for nonlinear suspension damper passive system are:

$$\ddot{x}_s = -\frac{1}{m_s} \left[k_s (x_s - x_u) + c_1 (\dot{x}_s - \dot{x}_u) + c_2 (\ddot{x}_s - \ddot{x}_u)^2\right]$$

$$\ddot{x}_u = -\frac{1}{m_u} \left\{k_s (x_s - x_u) + c_1 (\dot{x}_s - \dot{x}_u) + c_2 (\ddot{x}_s - \ddot{x}_u)^2 \right\} - k_s (q - x_s) - c_s (q - \dot{x}_s)$$

The sprung mass acceleration for linear passive and nonlinear suspension damper passive suspension system for random road excitation (q) are as shown in Fig. 11

Fig. 11 Sprung mass acceleration for linear passive and nonlinear suspension damper passive controller
The results are tabulated in Table 6

Table 6: Sprung Mass Acceleration for linear passive and nonlinear suspension damper

<table>
<thead>
<tr>
<th>Input</th>
<th>Controller</th>
<th>Sprung Mass Acceleration Range in m/s²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random Road Acceleration</td>
<td>Linear Passive</td>
<td>-5.00 to 4.49</td>
</tr>
<tr>
<td></td>
<td>Nonlinear suspension Damper Passive</td>
<td>-4.89 to 4.70</td>
</tr>
</tbody>
</table>

Result- It can be observed that due to suspension damper nonlinearity, the response deviates from linear system by 4.67% at upper side while 2.2% at lower side.

(f) Nonlinearity in tire damper

The damping in a pneumatic tire is mainly due to the hysteresis of tire materials. Generally, it is neither Coulomb nor viscous type. Its value is subject variation, depending on the design and construction of the tire, as well as operating conditions. The damping of pneumatic tires made of synthetic rubber compounds is considerably less than that provided by a shock absorber. From the measured data the tire damping coefficient is modeled as shown in Fig. 7.

![Fig. 12 Damping coefficient property for Hyundai Elantra 1992 car tire](image)

Therefore, tire damping force $f_{db}$ is modeled as third order polynomial function as,

$$f_{db} = \left( c_0 + c_1v + c_2v^2 + c_3v^3 \right) (\ddot{q} - \ddot{x}_n)$$

When
\[
\dot{q} - \ddot{x}_u < 0
\]

\[
f_{s_1} = 0
\]

When

\[
q - \ddot{x}_u \leq 0
\]

Where co-efficients are obtained from fitting the experimental data, which resulted in

c_3=0.011 \text{ N-s}^4/\text{m}^4 , c_2=0.896 \text{ N-s}^3/\text{m}^3 , c_1=-30.11 \text{ N-s}^2/\text{m}^2 \text{ and } c_0=15863 \text{ N-s/m.}

Therefore, the equations of motion for nonlinear tire damper passive system are:

\[
\ddot{x}_s = -\frac{1}{(m_s)} \left[ k_s (x_s - x_U) + c_s (\dot{x}_s - \dot{x}_U) \right]
\]

\[
\ddot{x}_u = -\frac{1}{(m_u)} \left\{ k_s (x_s - x_U) - [c_s (\dot{x}_s - \dot{x}_u)] - k_I (q - x_u) - (c_0 + c_1 v + c_2 v^2 + c_3 v^3)(\dot{q} - \dot{x}_u) \right\}
\]

The sprung mass acceleration for linear passive and nonlinear tire damper passive suspension system for random road excitation (q) are as shown in Fig.13

![Sprung mass acceleration for linear passive and nonlinear tire damper passive controller](image)

Fig. 13 Sprung mass acceleration for linear passive and nonlinear tire damper passive controller

The results are tabulated in Table 7
Table 7: Sprung Mass Acceleration for linear passive and nonlinear tire damper passive controller

<table>
<thead>
<tr>
<th>Input</th>
<th>Controller</th>
<th>Sprung Mass Acceleration Range in m/s²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random Road Acceleration</td>
<td>Linear Passive</td>
<td>-5.00 to 4.49</td>
</tr>
<tr>
<td></td>
<td>Non-linear Tire Damper Passive</td>
<td>-4.46 to 3.91</td>
</tr>
</tbody>
</table>

Result: It can be observed that due to tire damper nonlinearity, the response deviates from linear system by 12.91% at upper side while 10.8% at lower side.

III. RESULTS AND DISCUSSIONS

MATLAB SIMULINK is used as computer aided control system tool for modeling quarter car linear and nonlinear passive suspension system.

The simulation results obtained for these models are as shown in Fig. 3, Fig 5, Fig 7, Fig 9, Fig 11 and Fig 13. The comparison of all these is presented in Table 2, Table 3, Table 4, Table 5, Table 6 and Table 7 which shows the peak to peak acceleration of sprung mass.

From Fig. 3, Fig 5, Fig 7, Fig 9, Fig 11 and Fig 13 it is observed that, for linear passive controller minimum and maximum sprung mass acceleration is -5.00 m/sec² and 4.49 m/sec². These values for non-linear sprung mass passive controller, non-linear unsprung mass passive controller, non-linear suspension spring passive controller, non-linear tire spring passive controller, nonlinear suspension damper passive controller and nonlinear tire damper passive controller are -4.87 m/sec² to 4.39 m/sec², -4.99 m/sec² to 4.48 m/sec², -8.16 m/sec² to 6.47 m/sec², -4.88 m/sec² to 4.32 m/sec², -4.89 m/sec² to 4.70 m/sec² and -4.46 m/sec² to 3.91 m/sec² respectively.

IV. CONCLUSION

From the observation Tables and comparing the results, the degree of effectiveness has been observed more due to nonlinearity in spring stiffness. Maximum value of acceleration is observed due to nonlinearity in spring stiffness as compared to nonlinearities in mass and damper. Thus, it can be concluded that the nonlinearity in
stiffness’s is more significant than the nonlinearities in mass and damping. Therefore it is necessary to study the nonlinearity in spring stiffness in response analysis of vehicle dynamic system or the nonlinearity in spring stiffness can be linearised and response analysis is to be carried out.

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