LIGHT SCATTERING FROM A CLUSTER CONSISTS OF DIFFERENT AXISYMMETRIC OBJECTS

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ABSTRACT

Numerical results for random-orientation scattering matrices are presented for a cluster consists of a linear chain of different axisymmetric objects ensembles (spheres adhere spheroids). The calculation is based on a method that calculate the cluster T-matrix, and from which the orientation-averaged scattering matrix and total cross sections can be analytically obtained. Numerical results for the random-orientation scattering matrix are presented.

Keywords: Electromagnetic scattering, T- matrix method, Cluster of different particles.

1. INTRODUCTION

The particles that are formed in natural or in technological processes will possess complicated morphologies. Frequently, however, the morphological complexity of small particles arises from aggregation of individual particles that, by themselves, possess a simple shape. This aggregation of the particles constructs clusters. Fractal clusters, for example, are formed by different particles which aggregate and combined into sparse random fractal clusters [1]. There are many important applications for the light scattering by clusters. A model of light scattering for pollution identification and characterization by fractal clusters in the atmosphere, presents an important application for such type of the aggregated particles. A cluster of small particles, as a whole, is profoundly nondeterministic in shape, therefore the usual light scattering formulas based on regular particles (such as spheres, spheroids, cylinders) cannot, with reasonable accuracy, be directly applied to aggregated particles. However, if the individual particles making up the aggregate possess shapes that admit analytical solutions to the wave equations, then it is possible to calculate exactly, by appropriate superposition techniques, the radiative properties of the aggregate [2]. This approach has been well established for clusters of spheres [3], cluster of two prolate spheroids [4], clusters of fibres [5] and microscopic grains “dusting” of surfaces of larger host particles [6].

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In this paper, we modified the method introduced by Mackowski and Mishchenko [2] to determine the random-orientation scattering properties of a cluster of different shaped axisymmetric particles. Moreover, the calculation of light scattering properties of sphere clusters [7] is modified to be applicable for different cluster shapes. These modified techniques are combined with the previously published techniques [8, 9] to get a modified method that can be applied for general types of clusters.

Our aim is to exploit the scattering description of the $T$-matrix for a cluster consists of different axisymmetric particles, from which the random-orientation cross sections and scattering matrix can be obtained analytically. The advantage of the $T$-matrix method is that all properties of the scattering process can be contained. Thus with a computed $T$-matrix orientation the averaged scattering can be computed. It is also possible to compute multiple scattering by a number of neighboring particles by combining the $T$-matrices of the single constituents of the ensemble. To the best of our knowledge this is the first attempt to consider a cluster consists of more than three different shaped particles.

2. THEORETICAL ANALYSIS

The objective of this paper is calculating the scattered field from a cluster consists of different oriented particles and illuminated with a plane wave. The cluster and direction of the incident wave and scattered wave are shown in Fig.1. We assume that all the particles are axisymmetric, and $\mathbf{n}^{\text{inc}}$ and $\mathbf{n}^{\text{sca}}$ are the direction of the incident wave and the scattered wave respectively. The scattered field from a cluster consisting of $N_S$ axisymmetric particles is resolved into partial fields scattered from each particle in the cluster [2,3], i.e.,

$$\mathbf{E}_s = \sum_{i=1}^{N_S} \mathbf{E}_i$$  \hspace{1cm} (1)

where each partial field $\mathbf{E}_i$ is represented by an expansion of vector spherical harmonics (VSH) that are manipulated with respect to the origin of the $i^{th}$ axisymmetric object:

$$\mathbf{E}_i = H \sum_m \sum_n D_{mn} \left[ f^{i}_{\text{enn}} M^{3}_{\text{enn}}(\mathbf{r}) + f^{i}_{\text{oenn}} M^{3}_{\text{oenn}}(\mathbf{r}) + g^{i}_{\text{enn}} N^{3}_{\text{enn}}(\mathbf{r}) + g^{i}_{\text{oenn}} N^{3}_{\text{oenn}}(\mathbf{r}) \right]$$  \hspace{1cm} (2)

where $H$, and $D_{mn}$ are normalization factors, $M^{3}(\mathbf{r})$ and $N^{3}(\mathbf{r})$ are the VSH of the third kind (outgoing wave functions) obtained from the VSH of the first kind. The coefficients $f^{i}_{\text{enn}}$, $f^{i}_{\text{oenn}}$, $g^{i}_{\text{enn}}$ and $g^{i}_{\text{oenn}}$ are the scattered field expansion coefficients for the $i^{th}$ axisymmetric object. All the parameters and details of the analysis are given in [10-13].

The field arriving at the surface of the $i^{th}$ axisymmetric object consists of the incident field plus the scattered fields that originate from all other axisymmetric objects in the cluster. By use of the addition theorem for VSH the interacting scattered fields can be transformed into expansions of the field about the origin of $i^{th}$ axisymmetric object [2], which makes possible to formulate an analytical formulation of the boundary conditions at the surface. After truncation of the expansions to $n = N_{O,i}$ which is the maximum order retained for the individual axisymmetric object scattered field expansions, a system of equations for the scattering coefficients of $i^{th}$ axisymmetric particle can be constructed using the centered $T$-matrix of the cluster as follow [2,3],
\[
\begin{align*}
\begin{bmatrix}
  f_{i_{mn}}^j \\
  f_{i_{omn}}^j \\
  g_{i_{emn}}^j \\
  g_{i_{omn}}^j 
\end{bmatrix} &= \sum_{j = 1}^{N_j} \sum_{n' = 1}^{N_{n'}} \sum_{m' = -n'}^{n'} T_{ij}^{mn} m' n' \\
\begin{bmatrix}
  a_{em'n'}^j \\
  a_{om'n'}^j \\
  b_{em'n'}^j \\
  b_{om'n'}^j 
\end{bmatrix}
\end{align*}
\]

where \( N_j \) is the number of particles, \( n', n \) are the mode numbers and \( m', m \) are the azimuthal mode orders (\( n' = n \) for spheres, \( m' = m = 1 \) for on-axis incident plane wave and \( m' = m \) for an axisymmetric particle). Typically, \( N_{Oi,i} \) will be nonlinear proportional to the size parameter \( x_i \) of the individual axisymmetric object, \( f_{i_{mn}}^j, g_{i_{emn}}^j \) are the scattered field expansion coefficients for the \( i^{th} \) axisymmetric particle, and \( a_{mn}, b_{mn} \) denote the expansion coefficients for the incident field as described in [12, 13], \( e \) and \( o \) refer to even and odd respectively. Yet to describe the differential scattering cross sections (i.e., the scattering matrix), it is advantageous to transform the particle-centered \( T^{ij} \)-matrix into an equivalent cluster centered \( T \)-matrix that is based upon a single origin of the cluster. This transformation is given as [2,3]:

\[
T_{nl}^{ij} = \sum_{i=1}^{N_i} \sum_{j=1}^{N_j} \sum_{n=1}^{N_n} T_{i_{mn}}^{ij} j_{n1}^{ol} \cdot T_{nl1}^{ij} j_{n1}^{ol} 
\]

The \( J^{ij} \) and \( J^{Jo} \) matrices are based on the spherical Bessel function. The \( T \)-matrix given in the above equation is completely equivalent to those calculated with extended boundary condition methods [9, 10]. Other calculations of the additional coefficients are discussed in [2]. All the parameters and details of the analysis are given in [2,3]. Note that, the orientation averaged scattering matrix elements can be analytically obtained from the \( T \)-matrix for axisymmetric scatterers using the procedures developed in [10], and [13]. Since the \( T^{ij} \)-matrix can be calculated, then total cross sections in both fixed and random orientation can be obtained by performing, operations directly on \( T^{ij} \)-matrix.

The Stokes parameters \( I, Q, U \) and \( V \) which define the relation between the incident and scattered light are specified with respect to the plane of the scattering direction [10,13]. The transformation of the Stokes parameters upon scattering is described by the real valued 4x4 Stokes scattering matrix \( S \). Although each element of the scattering matrix depends on the scattering angle \( \Theta^{sca} \), there is no dependence on the azimuthal scattering angle \( \Phi^{sca} \) for the collections of identical randomly oriented particles considered here. For a collection of randomly oriented particles, the scattering matrix reduces to [13]:

\[
\begin{bmatrix}
  I^{sca} \\
  Q^{sca} \\
  U^{sca} \\
  V^{sca} 
\end{bmatrix} = \begin{bmatrix}
  S_{11}(\Theta) & S_{21}(\Theta) & 0 & 0 \\
  S_{21}(\Theta) & S_{22}(\Theta) & 0 & 0 \\
  0 & 0 & S_{33}(\Theta) & S_{34}(\Theta) \\
  0 & 0 & -S_{34}(\Theta) & S_{44}(\Theta) 
\end{bmatrix} \begin{bmatrix}
  I^{inc} \\
  Q^{inc} \\
  U^{inc} \\
  V^{inc} 
\end{bmatrix}
\]
The elements of the scattering matrix can be used to define specific optical observables corresponding to different types of polarization state of the incoming light. For example, if the incident radiation is unpolarized, then the $(1,1)$ element characterizes the angular distribution of the scattered intensity in the far-field zone of the target, while the ratio $-S_{21}(\Theta)/S_{11}(\Theta)$ gives the corresponding angular distribution of the degree of linear polarization. If the incident radiation is linearly polarized in the scattering plane, then the angular distribution of the cross-polarized scattered intensity is given by \( \frac{1}{2} [S_{11}(\Theta) - S_{22}(\Theta)] \) as described in [10, 13]. All of the $S_{ij}$ can be written in terms of $S_1$, $S_2$, $S_3$, $S_4$. For example, $S_{3\parallel} = -\text{Im}(S_1^* S_2 - S_2^* S_1)$, where the scattering matrix relating the Stoke's parameter has its basis in the following amplitude scattering matrix,

\[
\begin{bmatrix}
E'_1 \\
E'_\parallel
\end{bmatrix} = e^{ikr} \begin{bmatrix}
S_2 & S_3 \\
S_4 & S_1
\end{bmatrix} \begin{bmatrix}
E_1 \\
E_\parallel
\end{bmatrix}
\]

where $kr$ is the argument of the vector spherical wave function, $k=2\pi/\lambda$ is the wave number, $r$ is the position vector, and $i=\sqrt{-1}$. The incident field has been evaluated at $z=0$. $E_\parallel$ is the electric field component polarized parallel to the $X$-$Z$ scattering plane, $E_\perp$ is the electric field component polarized perpendicular to the $X$-$Z$ scattering plane, and $S_1$= the $\perp$ scattered field amplitude for $\parallel$ incident. $S_2$= the $\parallel$ scattered field amplitude for $\parallel$ incident. $S_3$= the $\parallel$ scattered field amplitude for $\perp$ incident. $S_4$= the $\perp$ scattered field amplitude for $\parallel$ incident.

![Scattering geometry](image)

**Fig.1.** Scattering geometry

3. NUMERICAL RESULTS

The procedure for calculating the $T$-matrix of a cluster is started with calculating the matrix $T'$ of the axisymmetric centered particle by using the code described in [8, 9], followed by merging the $T'$ into the cluster $T$-matrix through Eq (4). Finally, calculating the random orientation scattering matrix expansion coefficients using Eq (5) can be performed. The random orientation scattering matrix described in [2] and its code in [7] is modified here to deal with not only spheres but also with different shapes of axisymmetric objects in the cluster. The veracity of that modified technique is proven by several tests.
In the following we present comprehensive examinations of the scattering properties of different clusters using the presented modified technique along with other techniques presented in previous publications in [2, 3, 8, 9, 10, 13] for cluster of different special cases. For the present purposes we illustrate a small sample of results and point out some salient features. Four examples of different clusters are presented. First, a linear chain of three spheres in a cluster as illustrated in Fig. 2 and presented in [2]. Second, a linear chain of three particles in a cluster consists of a sphere in the center and an oblate spheroid in both sides of the sphere, as illustrated in Fig. 3. Third, a linear chain of three particles in the cluster consists of a sphere in the center and a prolate spheroid in both sides of the sphere as in Fig. 4. Last example is a linear chain of five particles in the cluster that contain a sphere in the center and two particles (an oblate and a prolate spheroid) in both sides of the sphere as illustrated in Fig. 5.

Some published cases in the literature are recomputed to confirm the performance of the presented technique. First the cluster shown in Figs. 2-5 is considered with relative refractive indices of the particles located at sides of the centered particle are assigned to one, i.e. the cluster becomes a centered sphere alone. Also the case of a single oblate or prolate off-centered spheroidal particle is recomputed when the relative refractive indices of certain particles in the linear chain cluster, in Figs. 3 and 4 are assigned to unity. The results of the angular scattering intensities are identical with the corresponding results published in the literature [8, 10, 13].

More tests are performed for different types of clusters to compare the results with those published in [2]. The test in this category is a cluster consists of three sphere, as in Fig. 2 having the same radius \( r=0.7957747 \mu m \), and relative refractive index of \( m=1.5+0.005i \). The computed results using our modified technique are typical with results in [2] as shown in Fig. 6. Finally as a rigorous test the two spheres in both side are deformed to have an axial ratio gradually decreased to 0.9, 0.8
and 0.7 to represent different clusters cases. A summary of the parameters of each constructed cluster is shown in table 1. The results of scattering matrix elements as a function of scattering angle $\Theta$ are shown also in Fig. 6. The results for each of these cases are changed gradually as the axial ratio changed from 1 to 0.7 which gives logical interpretation.

Table 1: parameters of different types of cluster consists of three particles with three different cases

<table>
<thead>
<tr>
<th>Number of particles (Ns)</th>
<th>Axial ratio (a/b)</th>
<th>Size parameter (x)</th>
<th>Refractive index (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear chain consists of three spheres</td>
<td>The three spheres has the same axial ratio: $a/b=1$ $r=0.7957747\mu m$</td>
<td>The three spheres has the same size parameter: $x=5$</td>
<td>The three spheres have the same refractive index: $m=1.5+0.005i.$</td>
</tr>
<tr>
<td>Linear chain consists of three particles, a sphere at the center of the cluster and two identical oblates, one at each side of the sphere.</td>
<td>Case 1: The sphere of: $r=0.7957747\mu m,$ Two oblate spheroids of the same axial ratio, $a/b=0.9,$ $b=0.7957747\mu m,$ $a=0.7161972\mu m.$</td>
<td>The sphere $x=5.$ Two oblate spheroids of the same size parameter $x=4.5.$</td>
<td>The three particles have the same refractive index: $m=1.5+0.005i.$</td>
</tr>
<tr>
<td></td>
<td>Case 2: The sphere of: $r=0.7957747\mu m.$ Two oblate spheroids of the same axial ratio, $a/b=0.8,$ $b=0.7957747\mu m,$ $a=0.6366197\mu m.$</td>
<td>The sphere $x=5.$ Two oblate spheroids of the same size parameter $x=4.$</td>
<td>The three particles have the same refractive index: $m=1.5+0.005i.$</td>
</tr>
<tr>
<td></td>
<td>Case 3: The sphere of: $r=0.7957747\mu m.$ Two oblate spheroids of the same axial ratio, $a/b=0.7,$ $b=0.7957747\mu m,$ $a=0.5570422\mu m.$</td>
<td>The sphere $x=5.$ Two oblate spheroids of the same size parameter $x=3.5.$</td>
<td>The three particles have the same refractive index: $m=1.5+0.005i.$</td>
</tr>
</tbody>
</table>
Fig. 6. Orientation-averaged scattering matrix elements for a linear chain of three particles as shown in Fig. 3 with different three cases as illustrated in table 1.
The second considered case is formed from a sphere of \( r = 0.7957747\, \mu m \), \( m = 1.5 + 0.005i \), at the center of a cluster and two prolate spheroids one at each side of the sphere as shown in Fig.4. The axial ratio of the two prolate spheroids is changed to 1.1, 1.2 and 1.3 to represent three different cases of clusters. A summary of the parameters for each case are shown in Table 2. The results of the scattering matrix elements corresponding to each case are illustrated in Fig. 7. A quick glance at both figures 6, 7 reveals that the configuration of the particles (spheroids adhere sphere) have a significant effect on the scattering properties of the cluster. The value of the scattering matrix element \( S_{11} \) in the forward direction attain a form that is nearly independent of the axial ratio of the spheroids located in both sides of the centered sphere. In the other directions, values of \( S_{11} \) for the chains depend on the axial ratio of the spheroids. Also near the forward direction, \( S_{11} \) oscillates slowly compared to the values near to the backward direction when the axial ratio of the spheroid increases.

Table 2: parameters of different types of cluster as shown in Fig.4 with three different cases

<table>
<thead>
<tr>
<th>Number of particles ((N_s))</th>
<th>Axial ratio ((a/b))</th>
<th>Size parameter ((x))</th>
<th>Refractive index ((m))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear chain consists of three particles, a sphere at the center of the cluster and two identical prolates, one at each side of the sphere.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 1: The sphere of: ( r = 0.7957747, \mu m ). Two prolate spheroids of the same axial ratio, ( a/b = 1.1 ), ( b = 0.7957747, \mu m ), ( a = 0.8753521, \mu m ).</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>The sphere ( x = 5 ). Two prolate spheroids of the same size parameter ( x = 5.5 ).</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>The three particles have the same refractive index: ( m = 1.5 + 0.005i ).</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Case 2: The sphere of: ( r = 0.7957747, \mu m ). Two prolate spheroids of the same axial ratio, ( a/b = 1.2 ), ( b = 0.7957747, \mu m ), ( a = 0.9549296, \mu m ).</td>
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<td></td>
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</tr>
<tr>
<td>The sphere ( x = 5 ). Two prolate spheroids of the same size parameter ( x = 6 ).</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>The three particles have the same refractive index: ( m = 1.5 + 0.005i ).</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 3: The sphere of: ( r = 0.7957747, \mu m ). Two prolate spheroids of the same axial ratio, ( a/b = 1.3 ), ( b = 0.7957747, \mu m ), ( a = 1.0345071, \mu m ).</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>The sphere ( x = 5 ). Two prolate spheroids of the same size parameter ( x = 6.5 ).</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The three particles have the same refractive index: ( m = 1.5 + 0.005 ).</td>
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</tbody>
</table>
Fig. 7. Orientation-averaged scattering matrix elements for a linear chain of three particles, as shown in Fig. 4 with different three cases as illustrated in table 2.
The last case considered here is that a cluster formed from five particles, one sphere in the center, and an oblate and a prolate spheroid in both sides of the sphere as shown in Fig. 5. A summary of the parameters are illustrated in table 3. The results of the scattering matrix elements are presented in Fig. 8. As shown in the figure the ripples of the scattering matrix elements are lower and smoother than in the previous illustrated cases.

**Table 3: Parameters of a Cluster as Shown Fig.5**

<table>
<thead>
<tr>
<th>Number of particles ($N_S$)</th>
<th>Axial ratio ($a/b$)</th>
<th>Size parameter ($x$)</th>
<th>Refractive index ($m$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear chain consists of five particles, one oblate spheroid and one prolate spheroid are located at each side of a centered sphere.</td>
<td>The sphere of: $r=0.7957747\mu m$. The two oblate spheroids are identical of an axial ratio, $a/b=0.7$, $b=0.7957747\mu m$, $a=0.5570422\mu m$. The two prolate spheroids are identical of an axial ratio, $a/b=1.3$, $b=0.7957747\mu m$, $a=1.0345071\mu m$.</td>
<td>The sphere $x=5$. The two oblate spheroids of the same size parameter $x=3.5$. The two prolate spheroids of the same size parameter $x=6.5$.</td>
<td>The five particles have the same refractive index: $m=1.5+0.005i$.</td>
</tr>
</tbody>
</table>
Fig. 8. Orientation-averaged scattering matrix elements for a linear chain of five particles, as shown in Fig. 5 with parameters as illustrated in table 3.
4. CONCLUSIONS

A technique is modified and developed in this paper to determine the random-orientation scattering properties of a cluster consists of different objects. Clustering particles affect the scattering by two mechanisms: far-field wave interference and near-field interactions, (or, equivalently, multiple scattering). We present a sample of results and illustrate some salient features of the scattering from clusters. The scattering matrix elements, as a function of scattering angle $\Theta_{\text{sc}}$, for three different clusters are computed and presented. The first case is a cluster formed by a sphere at the center and one oblate spheroid at both sides of the center sphere. The second case is performed by a sphere at the center of the cluster and one prolate spheroid at both sides of the center sphere. Values of $S_{11}$ in the forward direction show that, the matrix elements for the first and second cases are independent of the axial ratio of the spheroids. Whereas the values of $S_{11}$ near the backward directions, dependent on the axial ratio of the spheroids. Moreover a case of a cluster formed by five particles, one sphere in the center and an oblate and a prolate spheroids located in both sides of the sphere is investigated. The computed results show that the ripples in the matrix elements are lower and smoother than those of the previous considered cases (clusters of three particles).

REFERENCES

